# A Hybrid Classical Quantum Computing Approach to the Satellite Mission Planning Problem

Nils Quetschlich\*

Vincent Koch<sup>†</sup>

Lukas Burgholzer\*

Robert Wille\*‡

\*Chair for Design Automation, Technical University of Munich, Germany

<sup>†</sup>Technical University of Munich, Germany

<sup>‡</sup>Software Competence Center Hagenberg GmbH (SCCH), Austria

nils.quetschlich@tum.de v.koch@tum.de

lukas.burgholzer@tum.de

robert.wille@tum.de

https://www.cda.cit.tum.de/research/quantum

Abstract—Hundreds of satellites equipped with cameras orbit the Earth to capture images from locations for various purposes. Since the field of view of the cameras is usually very narrow, the optics have to be adjusted and rotated between single shots of different locations. This is even further complicated by the fixed speed-determined by the satellite's altitude-such that the decision what locations to select for imaging becomes even more complex. Therefore, classical algorithms for this Satellite Mission Planning Problem (SMPP) have already been proposed decades ago. However, corresponding classical solutions have only seen evolutionary enhancements since then. Quantum computing and its promises, on the other hand, provide the potential for revolutionary improvement. Therefore, in this work, we propose a hybrid classical quantum computing approach to solve the SMPP combining the advantages of quantum hardware with decades of classical optimizer development. Using the Variational Quantum Eigensolver (VQE), Quantum Approximate Optimization Algorithm (QAOA), and its warm-start variant (W-QAOA), we demonstrate the applicability of solving the SMPP for up to 21 locations to choose from. This proof-of-concept—which is available on GitHub (https://github.com/cda-tum/mqt-problemsolver) as part of the Munich Quantum Toolkit (MQT)-showcases the potential of quantum computing in this application domain and represents a first step toward competing with classical algorithms in the future.

# I. INTRODUCTION

Satellites orbiting the Earth have become a critical tool for a wide range of applications, including environmental monitoring, navigation, as well as communication. These satellites are usually equipped with cameras to capture images of the Earth's surface. However, this comes with a significant challenge: Satellites have limited time and resources to capture images, and decisions must be made what areas of the Earth to coverrequiring sophisticated algorithms to efficiently and effectively allocate resources and plan the routing of the satellite.

This Satellite Mission Planning Problem (SMPP) is a computationally challenging combinatorial optimization problem that involves scheduling satellite resources to obtain images of the Earth's surface. So far, the SMPP has been approached by utilizing classical algorithms—both using heuristics (e.g., simulated annealing and tabu search [1]) and exact approaches (e.g., linear integer programming [2] or branch-and-bound [3]). While these works have already been proposed decades ago, more recent papers propose extensions of the SMPP focusing on additionally objectives, e.g., considering also transmission tasks [4] or multiple satellites [5], [6]. Although a wide range of classical approaches for the SMPP and related problems have been developed, their potential for further improvements is rather small—especially, because the classical computing hardware development has shifted from revolutionary enhancements decades ago toward rather small, incremental enhancements.

Quantum computing, on the contrary, has provided tremendous improvements especially in recent years—both in software and hardware. Meanwhile, dedicated workflows such as those proposed in [7], [8] are available that allow solving problems that are computationally hard for classical algorithms. Especially, hybrid classical quantum algorithms which combine classical and quantum computing have shown promising results in solving various problems, such as in finance [9], optimization [10], chemistry [11], and physics [12]. Even a quantum annealing approach to solve the SMPP has already been proposed in [13].

In this paper, we propose a hybrid classical quantum approach for the SMPP. Therefore, the problem is formulated as a Quadratic Unconstraint Binary Optimization (QUBO) problem and encoded into a quantum circuit based on the Variational Quantum Eigensolver (VQE, [14]), the Quantum Approximate Optimization Algorithm (QAOA, [15]), and its warm-start variant (W-QAOA, [16]). The resulting circuits are then executed on a noise-free and a noise-aware quantum simulator. Evaluations show that the proposed approach can solve the SMPP for up to 21 possible images in reasonable time with often (close-to) optimal results—demonstrating the potential quantum computing offers in this application domain. All implementations are available on GitHub (https://github. com/cda-tum/mqt-problemsolver) as part of the Munich Quantum Toolkit (MQT). Combined with the anticipated trend of improvements for both quantum hardware and software, this approach may become a serious competitor for the currently used classical solvers in the future.

The remainder of this work is structured as follows. Section II reviews the SMPP in more detail and describes the hybrid classical quantum approach proposed in this work. The applicability of the resulting solution has been confirmed by experimental evaluations whose results are summarized in Section III. Finally, Section IV discusses the lessons learned before the paper is concluded in Section V.

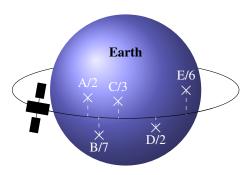


Fig. 1: Satellite Mission Planning Problem.

# II. SATELLITE MISSION PLANNING PROBLEM: HYBRID CLASSICAL QUANTUM APPROACH

In this section, the SMPP is described and formulated based on a more comprehensive problem description proposed in [13]. Thereafter, all the steps required to derive a quantum computing-based solution are described.

### A. Problem Description

The Satellite Mission Planning Problem (SMPP) refers to the complex task of selecting a subset of feasible locations from a pre-determined list of locations to-be-captured by a satellite. Each captured location is associated with a value. As a result of the narrow field of view of the satellite, its optics have to be adjusted and rotated between single shots of different locations—resulting in a time-consuming process referred to as "rotation time". This is even further complicated by the fixed speed of the satellite (determined by the satellite's altitude) which leads to a fixed "transition time" between the capture positions of locations (for the sake of simplification, it is assumed that each location can only be captured when the satellite is at its closest distance).

**Example 1.** Assume that an imaging satellite orbits the Earth at a given altitude with constant speed as depicted in Fig. 1 in black. In this setup, five to-be-captured locations are givendenoted by A to E with their respective associated value if captured. The goal is to determine a feasible subset of locations that maximizes the sum of their associated values. In this example, there are two infeasible pairs of locations (A/B and B/C) for which the rotation time is larger than the transition time. Consequently, A and C or only B can be selected in addition to D and E, which are feasible in both cases. Since the associated value of B is higher than both A's and C's values combined, the optimal selection is  $\{B, D, E\}$ .

More formally, the SMPP can be described as a binary constrained optimization problem:

$$\max_{x \in \{0,1\}^n} \sum_{i=1}^n x_i v_i \text{ with}$$

$$R(i,j) \le T(i,j) \text{ if } x_i = x_j = 1$$

$$(1)$$

with

• x being the vector of binary variables  $x_i$  representing whether a location i has been selected for imaging at its respective capture position  $s_i$  on the satellite orbit,

- $v_i$  being the associated value of a captured location i,
- R(i, j) being the required rotation time between two locations defined by the necessary satellite optics rotation time between angle  $a_i$  at satellite position  $s_i$  and angle  $a_j$  at position  $s_j$  divided by the constant optics rotation speed  $v_r$ :  $\frac{|a_i - a_j|}{v_r}$ , and • T(i,j) being the transition time between two capture
- positions  $s_i$  and  $s_j$  divided by satellite speed  $v_s$ :  $\frac{|s_i s_j|}{v_s}$ .

To derive a quantum computing-based solution to this problem, the general workflow proposed in [7] is followed: First, a suitable quantum algorithm is selected based on the problem to be solved. Subsequently, the problem must be encoded into a quantum circuit based on that selected algorithm. This quantum circuit is then executed and the resulting histogram is decoded to extract the actual problem solution.

# B. Quantum Algorithm Selection

The currently available quantum algorithms can be classified into two categories. The first class, referred to as *pure* quantum algorithms, has been proven to outperform its classical counter parts, such as Shor's [17] for factorization, Grover's [18] for unsorted database search, as well as the Harrow-Hassadim-Lloyd (HHL, [19]) algorithm for solving linear systems of equations. However, these algorithms generally assume close-to perfect quantum hardware which is not yet available to the necessary extent.

Hybrid classical quantum algorithms or the so-called Variational Quantum Algorithms (VQAs) are the second class of quantum algorithms. Prominent representatives are the Variational Quantum Eigensolver (VQE, [14]) to determine the ground state of a given physical system and the Quantum Approximation Optimization Algorithm (QAOA, [15]) used for various optimization problems. In contrast to pure quantum algorithms, the requirements regarding the quality of quantum hardware are less strict and, hence, these algorithms are frequently applied in the current Noisy-Intermediate-Scale-Ouantum (NISO, [20]) era. The general idea behind VOAs is to encode a given problem into a parameterized quantum circuit and utilize a classical optimizer to determine the most suitable parameter values so that the resulting quantum circuit allows one to determine the solution to the given problem.

To also apply VQAs to the SMPP, the constrained binary problem must be re-formulated to be of the form of a Quadratic Unconstrained Binary Optimization (QUBO) problem since that is the common input format for VQAs. Therefore, a penalty term is introduced to substitute the condition to be satisfied in the binary constrained optimization problem formulation in Equation 1. This leads to

$$\max_{x \in \{0,1\}^n} \sum_{i=1}^n x_i v_i - p \cdot \sum_{i=1}^{n-1} \sum_{j=i}^n x_i x_j \cdot c(i,j)$$
 (2)

with

- p being the penalty value that enforces the constraints to be satisfied and
- c(i, j) being a pre-calculated Boolean variable which is 1 if R(i,j) > T(i,j) and 0 otherwise.

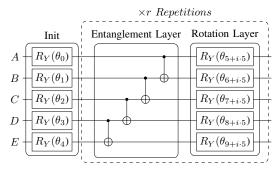


Fig. 2: SMPP encoded using VQE.

Using this QUBO formulation, VQAs can be utilized to solve the SMPP.

## C. Encoding as a Quantum Circuit

How a QUBO is translated into a quantum circuit depends on the selected quantum algorithm and the respective *encoding* of the problem formulation—stating a whole research area on its own with overviews given in [21]–[23]. For the SMPP, we used VQE, QAOA, and its *warm-start* variant W-QAOA to determine a feasible subset of locations from all possible ones with each location encoded by one respective qubit. The underlying circuit structures of those algorithms are described in the following.

The general quantum circuit structure of VQE is problem-independent and various ansatz functions are available. Here, a *RealAmplitudes*<sup>1</sup> ansatz with reverse linear entanglement has been chosen and is depicted in Fig. 2. It consists of the following building blocks:

- Init: A layer of parameterized single-qubit rotations.
- Entanglement Layer: The reverse linear entanglement ansatz is implemented based on a sequence of CNOT gates.
- Rotation Layer: Every repetition ends with another layer of single-qubit rotations implemented using  $R_Y$  gates. Each gate comes with its own rotation angle parameter.

Both the entanglement and rotation layers are repeated for r repetitions while the index i denotes the ith repetition.

The quantum circuit structure of QAOA comes with a significant difference: it is problem-dependent and consists of three building blocks as depicted in Fig. 3:

- State Preparation: Creates a superposition state of all qubits by applying *Hadamard* gates denoted by *H*.
- Cost Layer: Defines the SMPP instance by encoding weight factors into  $R_Z$  gates and infeasible location pairs as RZZ gates as depicted for the problem instance described in Example 1. Those  $\theta$  and  $\gamma$  factors are defined by the SMPP model describing its locations values and feasibility constraints among each other<sup>2</sup>.

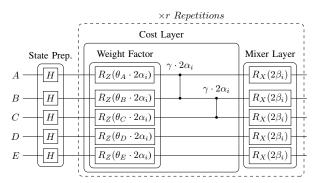


Fig. 3: SMPP encoded using QAOA.

 Mixer Layer: Enables the algorithm to theoretically be able to find the optimal parameter values and, therewith, a solution to the SMPP.

Again, both the cost and the mixer layers are repeated for arbitrarily many repetitions r with i denoting the ith repetition.

The resulting quantum circuits are both now capable of determining the SMPP solution in combination with a classical optimizer which is used for tuning the parameter toward an optimal solution. For that, their values can either be randomly initialized or, at least for QAOA, be pre-determined using its *warm-start* variant (W-QAOA, [16]).

#### D. Execution

While the generated quantum circuit could immediately be executed on a quantum simulator, the execution on the quantum device requires the quantum circuit to be *compiled* accordingly. Each quantum device induces constraints on the to-be-executed quantum circuit such as on the supported elementary quantum gates and, often, additionally on the connectivity of qubits. Various software tools have been proposed to automate the compilation in, e.g., [25], [26]. Thereafter, the quantum circuit is ready to be executed—resulting in a distribution of outcomes, also called a histogram, which comprises the determined solution. To extract it, the histogram must be decoded so that the actual solution is extracted. Although this step marks the end of the workflow for pure quantum algorithms, this differs for VQAs.

Here, as sketched in Fig. 4, the decoded solution is used to determine a *Cost* value which depends on the chosen parameter values of the parameterized quantum circuit and the problem itself.

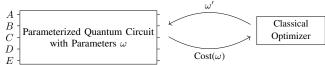


Fig. 4: VQA execution scheme.

This is also what enables VQE to determine solutions while the problem is not represented in the quantum circuit itself. The classical optimizer then adjusts the parameter values to evaluate their influence in the next iteration. With many of such iterations, the parameter values ideally converge to their optimal value. When this is the case, the algorithm terminates and the decoded solution of the quantum circuit executed with the optimal parameters is returned.

<sup>&</sup>lt;sup>1</sup>For details, see the Qiskit documentation at https://qiskit.org/documentation/stubs/qiskit.circuit.library.RealAmplitudes.html.

<sup>&</sup>lt;sup>2</sup>Note, that the  $\theta$  and  $\gamma$  are pre-determined before  $\alpha_i$  and  $\beta_i$  are optimized by transforming the QUBO problem into an *Ising Hamiltonian*—e.g., by using respective SDKs, such as Qiskit [24].

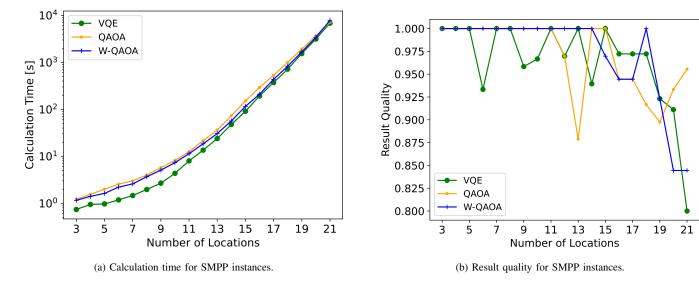


Fig. 5: Experimental evaluation for the satellite use-case using a noise-free simulation.

#### III. EXPERIMENTAL EVALUATION

In the following, the applicability of quantum computing to the SMPP is demonstrated. To this end, we first describe the evaluation setup and the problem instances considered. Subsequently, the results based on noise-free and noise-aware simulations are presented and discussed.

#### A. Setup

All implementations have been carried out using Qiskit [24] (v0.41.1) in *Python* and are available on GitHub (https://github.com/cda-tum/mqt-problemsolver) as part of the *Munich Quantum Toolkit* (MQT). To evaluate the proposed approach, SMPP instances from 3 up to 21 to-be-captured locations have been solved using VQE, QAOA, and W-QAOA in a noise-free simulation and up to 14 locatins in a noise-aware one. The problem instances themselves are generated as follows: Each image location is randomly placed with an arbitrary longitude while the latitude is restricted to be between -15 and 15 degrees (since the satellite is assumed to orbit around the equator and this condition ensures that each location is visible to the satellite) and is assigned a value of either 1 or 2.

For both, the noise-free and the noise-aware simulation, *Cobyla* is used as the classical optimizer. The maximal iterations are set to 100 for both optimizers in all evaluations. All VQAs are evaluated using 3 repetitions each. For VQE, a *RealAmplitudes* ansatz with reverse linear entanglement has been chosen.

To evaluate both the noise-free and the noise-aware simulations, the calculation time and result quality are evaluated.

#### B. Noise-free Simulation

Each problem instance is executed three times per algorithm and both the calculation time and the result quality are averaged to be more representative. All simulations are conducted using Qiskit's matrix-vector-based *Sampler* simulator. The resulting computation times for all three algorithms are provided in Fig. 5a. As expected, the computation time quickly increases

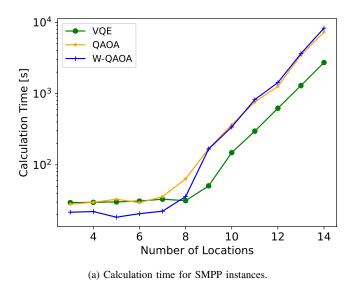
with the number of locations and shows an exponential trend (represented by the linear slope on a logarithmic scale) while VQE was significantly faster for smaller problem instances.

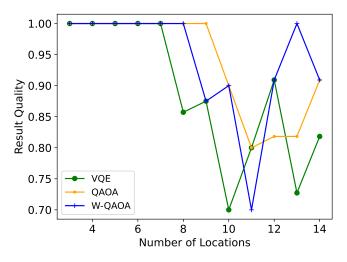
The respective result quality is shown in Fig. 5b. To this end, a classical computation of the optimal solution and the corresponding value is performed, which is then used as a baseline to measure the relative deviation of the VQAs—e.g., a result quality of 0.95 corresponds to a location selection which leads to an assigned accumulated value of 95% of the optimal one.

In general, there is a clear trend of degrading result quality with an increasing number of locations. Until around 12 locations, all algorithms have been able to determine the (close-to) optimal results with a result quality of > 90%. From there on, a notable decrease in result quality is occurring with VQE degrading the most—down to a result quality of 0.80 for the largest problem instance size of 21 locations.

## C. Noise-aware Simulation

noise-aware simulation is based on BackendSampler simulator using the FakeBackend for ibmq montreal with 27 qubits. The respective computation times for all algorithms are provided in Fig. 6a and follow a similar exponential trend for an increasing number of locations with W-QAOA being the fastest algorithm for smaller problem instances and VQE for larger ones. Compared to the noise-free simulation, the computation times are significantly higher for similar problem sizes. This is caused by a combination of three factors: the classical optimizer requires more iterations to converge at each execution, the additional compilation time to compile the underlying quantum circuit such that it is executable on the selected (but simulated) quantum device, and the increased simulation time for a noise-aware simulation. Due to this increase in computation time, the maximal number of locations evaluated is smaller compared to the noise-free simulation.





(b) Result quality for SMPP instances.

Fig. 6: Experimental evaluation for the satellite use-case using a noise-aware simulation.

The result quality is shown in Fig. 6b with a qualitatively similar trend of decreasing result quality for an increasing number of locations. However, the quality drop is greater with the lowest quality around 70%—already for a number of locations where the result quality was close-to optimal in the noise-free simulation.

Although the literature suggests that one uses the SPSA optimizer for noise-aware simulations (e.g., in [27]), that optimizer led to worse results while taking more computation time. The reason could be that the *loss landscapes* and the respective solutions of the considered SMPPs have been sufficiently distinct for the faster *Cobyla* optimizer not to degrade too much due to the induced noise.

Through the studies summarized above, the potential for solving the SMPP using VQAs has been evaluated for the first time. Based on that, the "lessons learned" are discussed in the following.

## IV. DISCUSSION

VQAs have demonstrated promising results in solving combinatorial optimization problems, including those in the satellite domain. However, they face several challenges that need to be addressed. One significant limitation is the requirement for a large number of qubits to solve larger problems (since each to-be-captured location is encoded as one qubit)—easily outgrowing what current simulators can reasonably simulate and the capabilities of available quantum devices. Furthermore, VQAs are known to suffer from the issue of barren plateaus, which can make finding the optimal solution difficult, particularly in complex optimization problems. Determining the optimal set of parameters for the classical optimizer is another challenge, particularly for large problems, and can increase the computational cost and time required for solving optimization problems.

Although classical solvers currently outperform VQAs, these hybrid classical quantum algorithms, in general, still present a promising approach since the ongoing development of quantum hardware and software is expected to improve

the performance of VQAs—making it an increasingly viable option for solving optimization problems in the future.

## V. CONCLUSIONS

Due to the increasing number of satellites and their limited resources, the Satellite Mission Planning Problem (SMPP) becomes more relevant. Classical solutions have already been proposed decades ago but have only seen evolutionary enhancements since then. Quantum computing, on the other hand, has the potential for a revolutionary improvement. To this end, a hybrid classical quantum computing approach to the SMPP is proposed in this work using three different Variational Quantum Algorithms: VQE, QAOA, and W-QAOA. To demonstrate their potential, SMPP instances have been experimentally evaluated in both a noise-free with up to 21 to-be-captured locations and a noise-aware simulation with up to 14 to-be-captured locations—often leading to (close-to) optimal results. This proof-of-concept showcases the general applicability of quantum computing to solve the SMPP and is available on GitHub (https://github.com/cda-tum/ mgt-problemsolver) as part of the Munich Quantum Toolkit (MOT). Although the proposed quantum approach cannot compete with the classical solutions yet, promising developments in both quantum hardware and software—especially in recent years—might change that in the future.

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