

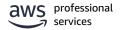
# The Search for Applications of Quantum Computing in Industry

qReduMIS: A Quantum-Informed Reduction Algorithm for the Maximum Independent Set Problem

Martin Schuetz Global Practice Lead Amazon Advanced Solutions Lab

### **Agenda**

- Introduction
- Overview: Challenge, Situation, Solution
- The MIS problem and Rydberg atom arrays
- Deep dive into qReduMIS algorithm
- Results: Performance analysis
- Summary and outlook



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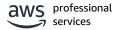
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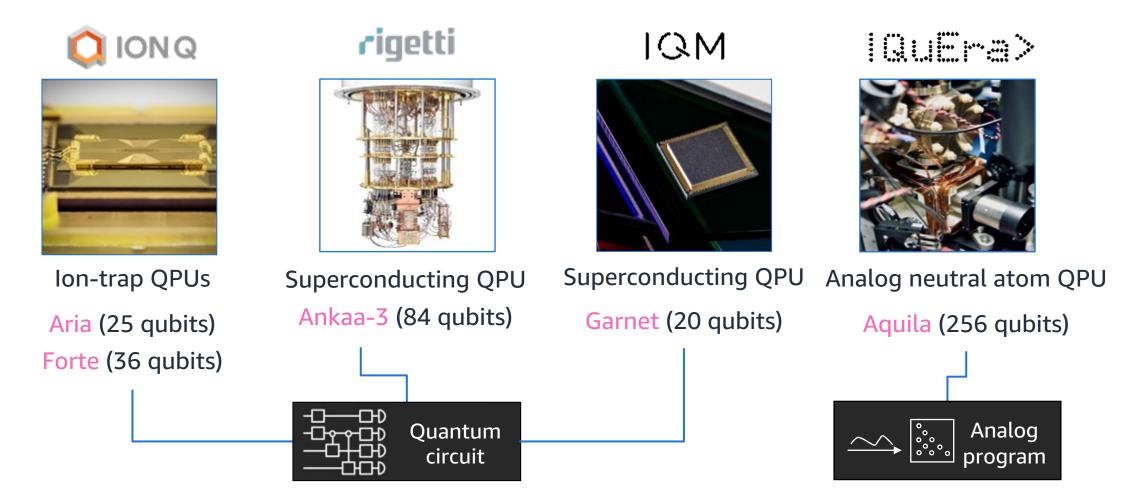


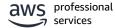
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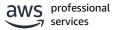




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#### Optimization of Robot Trajectory Planning with Nature-Inspired and Hybrid Quantum Algorithms

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(Dated: June 9, 2022)

arXiv:2206.03651

#### End-to-end resource analysis for quantum interior point methods and portfolio optimization

```
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arXiv:2211.12489

#### Explainable AI using expressive Boolean formulas

Gili Rosenberg,<sup>1</sup> J. Kyle Brubaker,<sup>1</sup> Martin J. A. Schuetz,<sup>1,2</sup> Grant Salton,<sup>1,2,3</sup> Zhihuai Zhu,<sup>1</sup> Elton Yechao Zhu,<sup>4</sup> Serdar Kadıoğlu,<sup>5</sup> Sima E. Borujeni,<sup>4</sup> and Helmut G. Katzgraber<sup>1</sup>

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(Dated: June 8, 2023)

arXiv:2306.03976

#### Hardness of the Maximum Independent Set Problem on Unit-Disk Graphs and Prospects for Quantum Speedups

Ruben S. Andrist,<sup>1,\*</sup> Martin J. A. Schuetz,<sup>1,2,\*</sup> Pierre Minssen,<sup>3,\*</sup> Romina Yalovetzky,<sup>3,\*</sup> Shouvanik Chakrabarti,<sup>3</sup> Dylan Herman,<sup>3</sup> Niraj Kumar,<sup>3</sup> Grant Salton,<sup>1,2,4</sup> Ruslan Shaydulin,<sup>3</sup> Yue Sun,<sup>3</sup> Marco Pistoia,<sup>3,†</sup> and Helmut G. Katzgraber<sup>1,†</sup>

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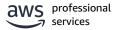
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(Dated: July 19, 2023)

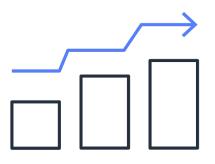
arXiv:2307.09442



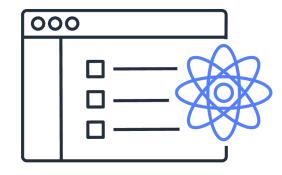
# Overview: Challenge, Situation, Solution



## The challenge





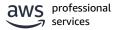


#### **Real-world optimization problems**

- Large scales (e.g., 1000+ assets)
- Constraints (e.g., budget constraints)
- Stringent time and quality requirements

#### **Near-term quantum computers**

- Small scales (hundreds of qubits)
- Unconstrained Hamiltonian encoding
- Hardware noise and other imperfections



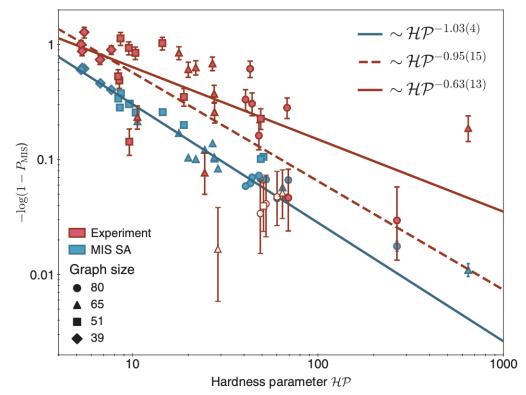
#### The situation around 2022

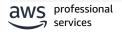
## **Quantum optimization of maximum independent set using Rydberg atom arrays**



Realizing quantum speedup for practically relevant, computationally hard problems is a central challenge in quantum information science. Using Rydberg atom arrays with up to 289 qubits in two spatial dimensions, we experimentally investigate quantum algorithms for solving the maximum independent set problem. We use a hardware-efficient encoding associated with Rydberg blockade, realize closed-loop optimization to test several variational algorithms, and subsequently apply them to systematically explore a class of graphs with programmable connectivity. We find that the problem hardness is controlled by the solution degeneracy and number of local minima, and we experimentally benchmark the quantum algorithm's performance against classical simulated annealing. On the hardest graphs, we observe a superlinear quantum speedup in finding exact solutions in the deep circuit regime and analyze its origins.







## Our collaboration with JPMorgan Chase

PHYSICAL REVIEW RESEARCH 5, 043277 (2023

#### Hardness of the maximum-independent-set problem on unit-disk graphs and prospects for quantum speedups

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Rydberg atom arrays are among the leading contenders for the demonstration of quantum speedups. Motivated by recent experiments with up to 289 qubits [Ebadi et al., Science 376, 1209 (2022)], we study the maximum independent-set problem on unit-disk graphs with a broader range of classical solvers beyond the scope of independence or problem on united agreeds with a bronder range of classical sobverts beyond the except of independence or problem or problem or problem. The problem of the results, we propose protocols to systematically tune problem hardness, motivating experiments with Rydbers atom arrays on instances orders of magnitude harder (for established classical solvers) than previously studie

#### I. INTRODUCTION

Combinatorial optimization problems are pervasive across science and industry, with prominent applications in ar-eas such as transportation and logistics, telecommunications, nufacturing, and finance. Given its potentially far-reaching impact, the demonstration of quantum speedups for prac-tically relevant, computationally hard problems (such as inatorial optimization problems) has emerged as one of

the greatest milestones in quantum information science.

Over the past few years, programmable Rydberg atom
arrays have emerged as a promising platform for the implementation of quantum information protocols [1–7] and (in
particular) quantum optimization algorithms [8–14]. Some of
the exquisite and experimentally demonstrated capabilities of these devices include the deterministic positioning of individual neutral atoms in highly scalable arrays with arbitrary arrangements [15,16], the coherent manipulation of the inte arrangements [15,16] the coherent manipulation of the inter-nal states of these atoms (including excitation into strongly excited Rydberg states) [17–19], the ability to coherently shuttle around individual atoms [20], and strong interactions mediated by the Rydberg blockade mechanism [12,21–23]. The physics of the Rydberg blockade mechanism has been shown to be intimately related to the canonical (NP-hard)

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maximum-independent-set (MIS) problem [8], in particular for unit-disk graphs. The MIS problem involves finding the largest independent set of vertices in a graph, i.e., the largest subset of vertices such that no edges connect any pair in the set; compare Fig. 1 for a schematic illustration. As shown in Ref. [8], MIS problems can be encoded with (effectivel two-level) Rydberg atoms placed at the vertices of the targe (problem) graph. Strong Rydberg interactions between aton then prevent two neighboring atoms from being simultane-ously in the excited Rydberg state, provided they are within the Rydberg blockade radius, thereby effectively implementthe Rydberg blockade radius, thereby effectively implementing the independence constraint underlying the MIS problem. By virtue of this Rydberg blockade mechanism, Rydberg atom arrays allow for a hardware-efficient encoding of the MIS problem on unit-disk graphs, with the (unable) disk radius  $R_6 \sim 1{-}10 \,\mu \mathrm{m}$  setting the relevant length-scale [6].

#### Overview of the main results

Recently, a potential (superlinear) quantum speedup ove classical simulated annealing was reported for the MIS prob lem [12] based on variational quantum algorithms run o Published by the American Physical Society under the terms of the Creative Common Attribution 4.0 International Genese Farber states of the Creative Common Attribution 4.0 International Genese Farber state-of-the-art classical solvers. Motivated by this experiment, we perform a detailed analysis of the MIS probler

#### PHYSICAL REVIEW RESEARCH 7 023142 (2025

#### Decomposition pipeline for large-scale portfolio optimization with applications to near-term

Atithi Acharya. Romina Yalovetzky . Pierre Minssen. Shouyanik Chakrabarti. Ruslan Shaydulin. Rudy Raymond. Aultin Acharya, Romina Valorestelyo, P. Perre Minners, "Shorwanik Chairbarth," Raulin Shuydhini, Rudy Ra, Visus Sun O', Djulin Heimer, Richen S., Andrew, "Grant Sallom O', "Manter J. A., Schwer, "Helmen G. Kazzy "Global Technology Applied Research, "Philosopic Classe, New York, New York 10017, USA "Assumer, Quantum Computing, Passadena, California 91125, USA "Atlast Center for Quantum Computing, Passadena, California 91125, USA "California Internal of Technology, Passadena, California 91125, USA "California Internal of Technology, Passadena, California 91125, USA

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ing, are often intractable or difficult to solve exactly. In this work, we propose and benchmark a decomposition speline targeting portfolio optimization and rebalancing problems with constraints. The nineline decomposition pipeline targeting portfolio optimization and rebalancing problems with constraints. The pipeline decomposes the optimization problem into constrained supplements with constraints. The pipeline decomposes the optimization problems into constrained supplements with a refuse to sheet separately and aggregated to give a final result. Our pipeline includes there main components: preprocessing of correlation matrices based on random matrix there, which could clustering based on Newman salgerithm, and risk rebalancing. Due empirical results show that our pipeline consistently decomposes rarel—world portfolio optimization problems into subspondens with an exclusion of approximately 90°s. Since unproblems are then solved independently supplements are then solved independently on the subspondens are then solved independently on the solved independently on the subspondens are then solved in the subspondens are the solved independently on the subspondens are the solved in the subspondens are th pipeline drastically reduces the total computation time for state-of-the-art solvers. Moreover, by decomposing large problems into several smaller subproblems, the pipeline enables the use of near-term quantum devices as solvers, providing a path toward practical utility of quantum computers in portfolio optin

#### L INTRODUCTION

Portfolio optimization (PO) plays a vital role in manag-ing vast amounts of global financial assets, requiring rapid and robust algorithms to effectively make decisions about investing capital, while managing risks. Often, PO problems include integer decision variables, which arise when the underlying assets are traded in discrete quantities. Moreover such optimization problems impose constraints on the deci-sion variables, such as having a fixed minimum and maximum number of selected assets, or a maximum total risk exposure of the portfolio.

of the portfolio.

Typically, PO problems can be formulated as mixedinteger programming (MIP) problems, for which a wide range
of methods have been proposed. In particular, various local
search methods have been proposed and successfully applied
[1–3]. However, these methods often have several drawbacks. when applied to MIP problems, such as scalability, the risk of thing trapped in local optima, and difficulty handling comex constraints. On the other hand, exact methods, such a use based on the branch-and-bound [4,5] search method, of the most widely used tools for many hard MIP problems.

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Notably, commercially available branch-and-bound based solvens offered by CPLEX [6] and GUOSHIT] have been widely adopted in the industry for optimization problems, such as traveling salesman, graph partitioning, and quadratic assignment problems [8]. These solvers employ the branch-and-out procedure, which effectively combines branch-and-bound with cutting-plane methods, and often reduces execution time. Despite their widespread success in solving practical prob Despite their widespread success in solving practical prob-lems of small and intermediate sizes, MIP problems are NP-hard in the worst case. Therefore, given the generic ex-ponential runtime scaling and ever-increasing problem sizes for real-world PO problems, it is crucial to develop techniques that can improve the scope and scale of problems these solvers were benefits.

PO with near-term quantum computing. In parallel to the development of classical optimizers, quantum computing has shown promise for providing computational speedups for some optimization problems of relevance to science and insome optimization problems of relevance to science and in-dustry [9-11], particularly in portfolio optimization problems [12,13]. A variety of quantum algorithms have been proposed, including well-studied quantum heuristics: like the quantum approximate optimization algorithm (QAOA) [14,15], which has been executed on quantum hardware for small-scale opti-mization problems [16-19], including portfolio optimization [20-23], with up to approximately 100 variables

Another approach to quantum optimization is quantum nnealing [3], which solves optimization problems by findin

#### Quantum Compilation Toolkit for Rydberg Atom Arrays with Implications for Problem Hardness and Quantum Speedups

Martin J. A. Schuetz, 1, 2, \* Ruben S. Andrist, 1, \* Grant Salton, 1, 2 Romina Yalovetzky, 3 Rudy Raymond, 3 Yue Sun,<sup>3</sup> Atithi Acharya,<sup>3</sup> Shouvanik Chakrabarti,<sup>3</sup> Marco Pistoia,<sup>3,†</sup> and Helmut G. Katzgraber<sup>1</sup> Amazon Adeanced Solutions Loh, Scattle, Washington 98170, USA

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(Dated: January 15, 2025)

We propose and implement a comprehensive quantum compilation toolist for solving the maximum independent of (MSI) problem on quantum machine based on Rytheya ground respect on early of the order-tood pipeline involves three core components to efficiently map generic MIS instances onto Rytherg arrays with unti-falsk connectivity, with modulos for graph reduction, hardware compatibility checks, and graph embedding. The first module (reducer) provides hardware-agnostic and deterministic reduction logic that iteratively reduces the problem size via lay crigate removals. We find that real-world networks can typically be reduced by orders of magnitude on sub-second tim scales, thus significantly cutting down the eventual load for quantum devices. Moreover, we show that reduction techniques may be an important tool in the ongoing search for potential quantum speedups, given their ability to identify hard problem instances. In particular, for Rydberg-native MIS instances, we observe signatures of an easy-hard-easy transition and quantify a critical degree indicating the onset of a hard problem regime. The second module (compatibility checker) implements a hard-wave compatibility checker that quickly determines whether or not a given input graph. ments a hardware compilability checker that inquickly determines whether or not a given input graph any be compatibility the criterious imposed by Ryldreg quantum hardware. The third mod-ule (embedder) describes hardware-efficient graph embedding routines to generate (approximate) encodings with controllable overhead and optimized and cling laberments. We exemplify our pipeline with experiments run on the QuEra Aquida device available on Amasom Braket. In aggregate, our work provides a set of tools that extends the class of problems that can be tacked with near-term work provides a set of tools that extends the class of problems that can be tacked with near-term

#### I. INTRODUCTION

The field of combinatorial optimization (CO) involves the search for the extremum of an objective function (such as a cost or profit value) within a finite (but usually large) set of candidate solutions. Despite often being simple to conceptualize, many CO problems are hard to solve and even NP-hard, representing some of the most exquisite, yet challenging computational problems known
[1, 2]. Given its close ties to the maximum clique, minimum vertex cover, and set packing problems [3], the maximum independent set (MIS) problem represents an important paradigmatic CO problem, with practical ap-plications in virtually any industry sector. These include network design [4], vehicle routing [5], and quantitative finance [6-8], to name a few.

Quantum optimization. Over the last few decades, quantum optimization (QO) has emerged as a novel aradism for solving such discrete ontimization problems [9], in pursuit of quantum speedups over the best-known classical algorithms for practically relevant problems. Prominent QO algorithms include quantum annealing algorithms (QAA) [10-14] as well as hybrid (quantum classical) algorithms such as the quantum approximate

In these approaches, typically the optimal solution to the classical optimization problem at hand is encoded in the

Optimization with Rydberg arrays. Over the last few years, analog neutral-atom quantum machines in the form of Rydberg atom arrays have emerged as a novel class of programmable and scalable special-pur quantum devices geared towards optimization workloads [16, 19–27]. In particular, recent experiments with up to 289 Rydberg atoms have reported a potential superlinear quantum speedup over classical simulated anneal ing for the MIS problem, with pioneering implementa-tions of both QAA and QAOA within the same exper-imental setup [22, 28]. Because of the isotropic nature of the Rydberg blockade mechanism [29–31], these first generation experiments were inherently limited to maximum independent set (MIS) problem instances on a minim independent set (Mis) proteem instances on a restricted class of geometric graphs known as unit-disk (UD) graphs [19, 20, 32]. Ultimately, however, the use-fulness of these devices will depend on the scope of pro-lems they can tackle beyond the hardware-native MIS-LIM-scale.

Embedding schemes. Embedding techniques are de signed to expand the scope of problems supported by a given quantum device by mapping logical input problems to physical representations compatible with connectivity-

#### qReduMIS: A Quantum-Informed Reduction Algorithm for the Maximum Independent Set Problen

Martin J. A. Schuetz, 1, 2, \* Romina Yalovetzky, 3, \* Ruben S. Andrist, 1 Grant Salton, 1, 2 Yue Sun, 3 Rudy Raymond, 3 Martin J. A. Scalied," "Rules I Scaling," Rules I S. Abdrils, 'Crait Salido," "The Sun,' Ruley Raymon, Shorwanik Chakrabardi," Attih Acharya, "Ruslan Basquini," Marco Piscio," and Helmut G. Katzgrabert. "Amazon Adenoced Solutions Lis, Scalife, Washington 89170, USA "AWS Center for Quantum Computing, Pasadem, CA 91155, USA "Global Technology Applied Research, JPMersyne Chass, New York, NY 10017 USA (Datel: March 18, 2025)

We propose and implement a quantum-informed reduction algorithm for the maximum indeper dent set problem that integrates classical kernelization techniques with information extracted fron quantum devices. Our larger framework consists of dedicated application, algorithm, and hard ware layers, and easily generalizes to the maximum weight independent set problem. In this hybrid quantum-classical framework, which we call qReduMIS, the quantum computer is used as a coessor to inform classical reduction logic about frozen vertices that are likely (or unlikely) to be in large independent sets, thereby opening up the reduction space after removal of targeted sub-graphs. We systematically assess the performance of qReduMIS based on experiments with up to 231 qubits run on Rydberg quantum hardware available through Amazon Braket. Our experiments show that qReduMIS can help address fundamental performance limitations faced by a broad set of (quantum) solvers including Rydberg quantum devices. We outline implementations of qReduMIS with alternative platforms, such as superconducting qubits or trapped ions, and we discuss potential

#### I. INTRODUCTION

The maximum independent set (MIS) problem is a paradigmatic, NP-hard combinatorial optimization problem with close ties to the well-known, complementary sem with close ties to the weit-known, complementary maximum clique and minimum vertex cover problems [1]. Practical applications can be found in virtually every in-dustry, including computer vision [2], map labeling [3], network design [4], vehicle routing [5], and quantitative finance [6-8], to name a few. Given an undirected graph G = (V, E) with nodes V and edges E, an independent se  $y = (V, \mathcal{E})$  with nones V and edges  $\mathcal{E}$ , an independent set is a subset  $T \subseteq V$ , such that no two vertices in the set T share an edge. The goal of the MIS problem is to find an independent set T with maximum cardinality. Such a set is called a maximum independent set of size  $|MIS| = |\mathcal{I}|$ . Similarly, for a weighted graph  $G = (V, E, \omega)$  with vertex weight function  $\omega : \mathcal{V} \to \mathbb{R}^+$ , the generalized maximum weight independent set (MWIS) problem asks for an independent set  $\mathcal{I}$  with maximum weight  $\omega(\mathcal{I}) = \sum_{v \in \mathcal{I}} \omega(v)$ .

Classical ReduMIS. One of today's leading heuristics for the MIS problem is the ReduMIS algorithm [9], which intermixes a suite of reduction (or kernelization) techniques with a heuristic evolutionary algorithm. Reduction techniques are polynomial time procedures that can shrink a given input graph to an irreducible kernel by removing well-defined subgraphs. These subgraphs are removed through targeted selection of exposed ver-tices that are provably part of some maximum(-weight) independent set, hence maintaining optimality [10-16]. Whenever an irreducible kernel is identified, ReduMIS

makes use of an evolutionary approach to identify and remove vertices that are likely part of a large independent set, thereby opening up yet again the reduction space for further kernelization. Ultimately, a solution to the MIS or MWIS problem on the original input graph can be found by undoing previously applied reductions, with

Quantum optimization. Over the last few decades, quantum optimization algorithms have emerged as a novel paradigm for solving combinatorial optimization problems, such as the MIS problem [18]. Prominent examples include quantum annealing algorithms (OAA) [19-23] and the quantum approximate optimization al gorithm (QAOA) [24, 25]. In particular, analog neutral-atom quantum machines in the form of Rydberg atom arrays have attracted broad interest as a novel class of programmable and scalable special-purpose quantum de the MIS (and MWIS) problem on unit-disk (UD) graphs the anis (and awn's) problem on unit-mask (UD) graphs 26–34l, Recent QAA-based experiments report on a po-tential super-linear quantum speedup over classical sim-ulated annealing [29, 35]. However, for a broad set of Markov-chain Monte Carlo algorithms, the same experiments also show that the algorithmic performance is sur ments asso snow trant the algorithmic periorizance is sup-pressed exponentially in the conductance-like hardness parameter  $\mathbb{H}$ , defined as  $\mathbb{H} = D_{[\mathrm{MMS}]-1}/[[\mathrm{MMS}]]$ , where  $D_{\alpha}$  denotes the despeneacy of the independent set of size  $\alpha$ . Specifically, the success probability to find the MIS in a single algorithmic run (shot), denoted as Page is shown to follow the form  $P_{cos} \approx 1 - \exp(-CH^{-})$ where  $\beta$  depends on details of the given algorithm, and C refers to a positive (fitted) constant.

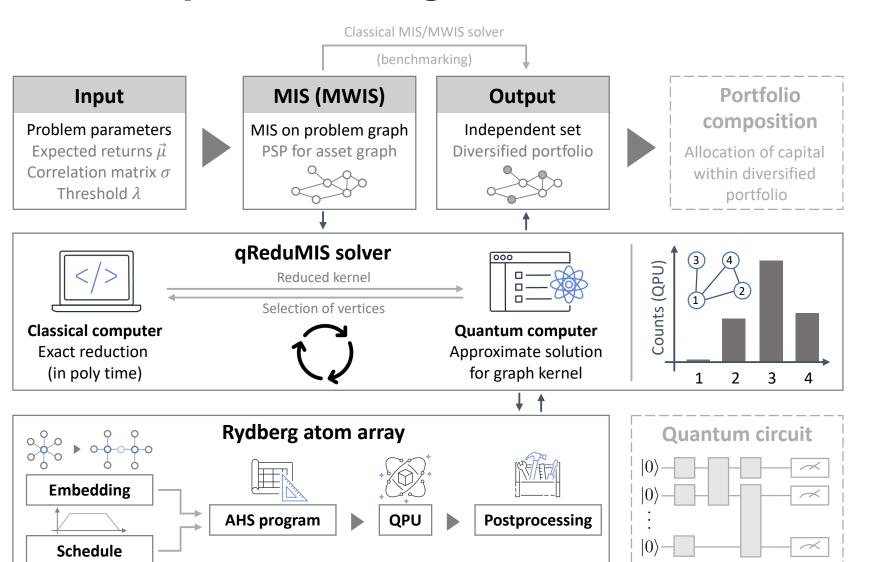
Quantum ReduMIS (gReduMIS). In this work we

- Study the hardness of the MIS-UD problem
- Problem native for Rydberg quantum devices

- Decomposition pipeline for large portfolio optimization problems
- Reduction by ~80% for real-world instances
- Compilation toolkit for Rydberg atom arrays
- Hardware-efficient embedding with optimized overhead
- **Ouantum-informed** qReduMIS algorithm
- Integration of exact classical reduction with QPU as co-processor



## The solution: qReduMIS algorithm



application

algorithm

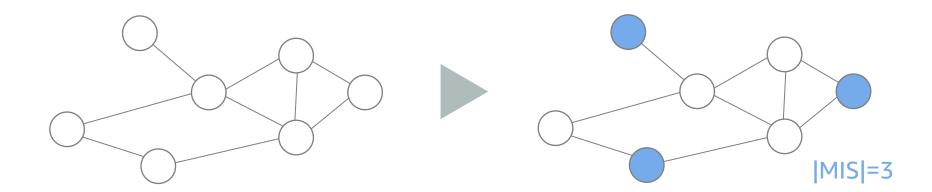
hardware

# The MIS problem and Rydberg atom arrays



#### The MIS problem

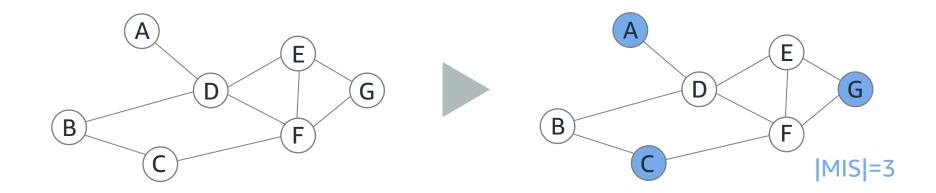
 Problem statement: Given an undirected graph, find largest independent set of vertices (i.e., the largest subset of vertices such that no edges connect any pair in the set).



- MIS problem dual to maximum clique and minimum vertex cover problems.
- Practical applications: Map labeling, network design, vehicle routing, quantitative finance, ...

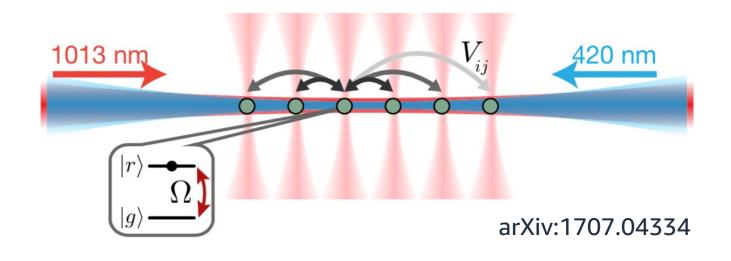
#### Portfolio selection

 Problem statement: Given a (large) universe of assets, find the largest set of uncorrelated assets.



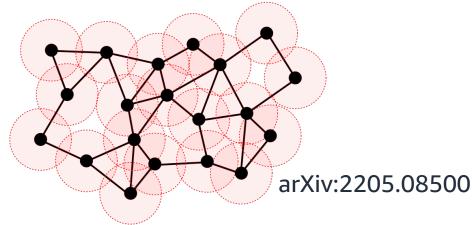
• Solution: Invest in 3 uncorrelated assets {A, C, G}, out of 7 assets.

## Rydberg atom arrays in a nutshell

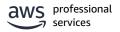


#### Effective spin Hamiltonian:

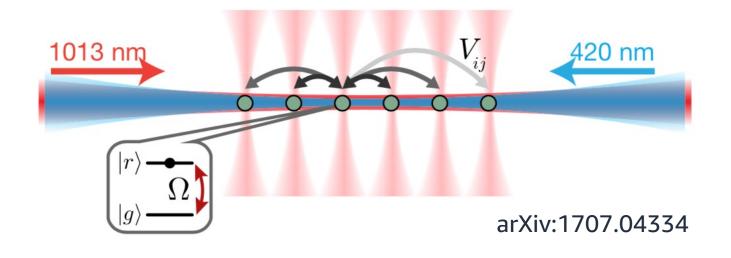
$$H = \sum_{i} \Omega \sigma_i^x - \Delta \sigma_i^z + \sum_{i < j} V_{ij} n_i n_j$$



- Customizable geometry of Rydberg atoms via trapping potentials
- Strong Rydberg interactions lead to a unit disk (UD) graph connectivity
- Dynamical control of the Hamiltonian parameters

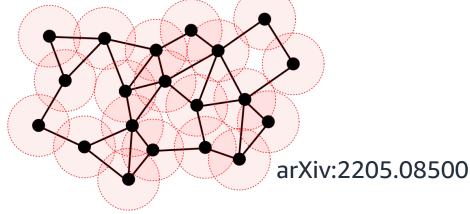


## Rydberg atom arrays in a nutshell

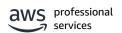


#### Quantum optimization:

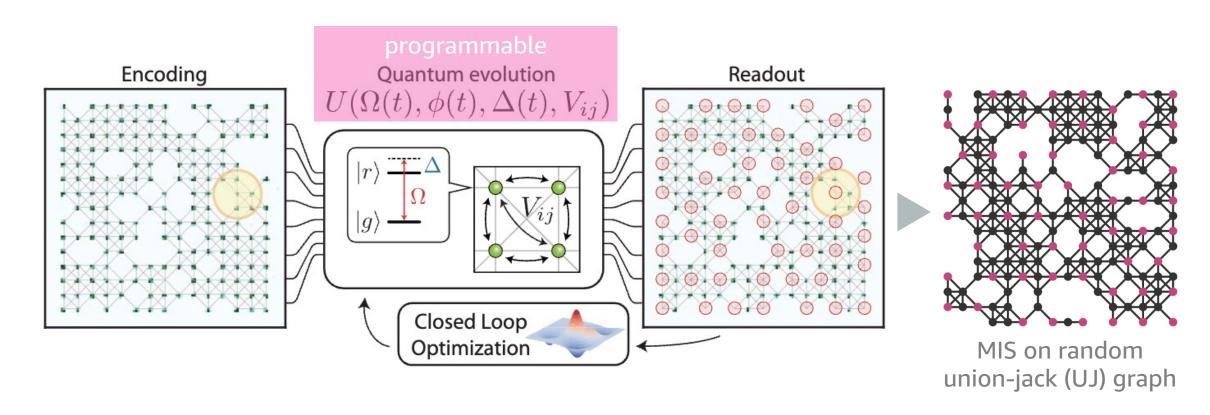
$$H = \sum_{\substack{i \\ \text{driver}}} \Omega \sigma_i^x - \Delta \sigma_i^z + \sum_{\substack{i < j \\ \text{MIS cost}}} V_{ij} n_i n_j$$



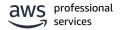
- Customizable geometry of Rydberg atoms via trapping potentials
- Strong Rydberg interactions lead to a unit disk (UD) graph connectivity
- Dynamical control of the Hamiltonian parameters



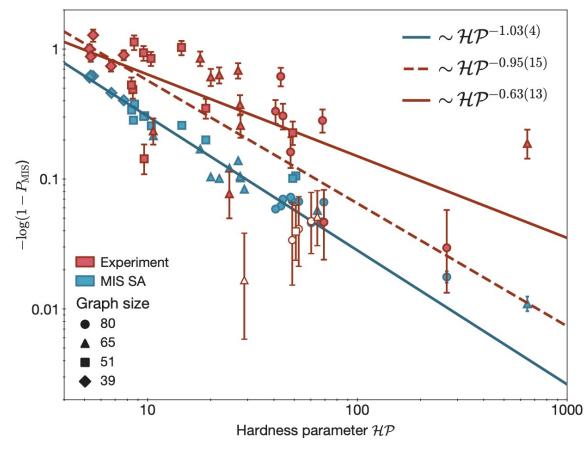
## Quantum optimization for MIS on random UJ graphs



Ebadi et al., arXiv:2202.09372 [Science **376**, 1209 (2022)]



## **Performance on random UJ graphs**



Ebadi et al., arXiv:2202.09372 [Science 376, 1209 (2022)]

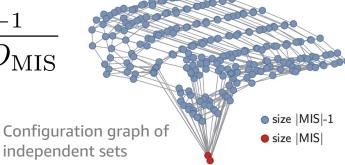
- Quantum algorithm reported to outperform classical simulated annealing (SA).
- Performance scaling:

$$P_{\mathrm{MIS}} \sim 1 - \exp\left(-C\mathbb{H}^{-1.03}\right)$$
 SA

$$P_{\rm MIS} \sim 1 - \exp\left(-C\mathbb{H}^{-0.63}\right)$$
 QA

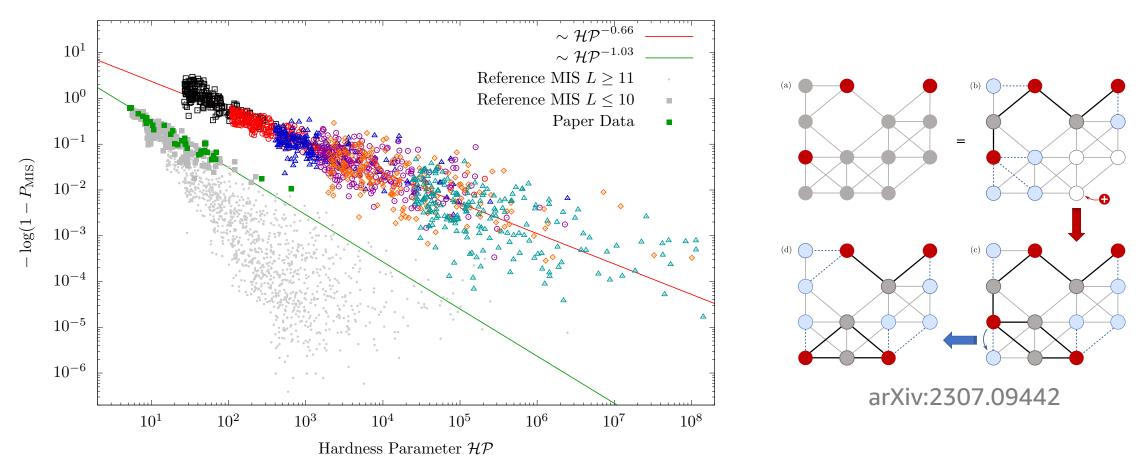
Hardness parameter:

$$\mathbb{H} = \frac{D_{\text{MIS}-1}}{|\text{MIS}| \cdot D_{\text{MIS}}}$$

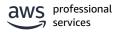




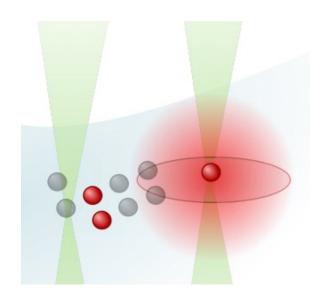
#### Results for native SA solver

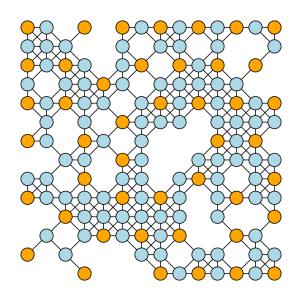


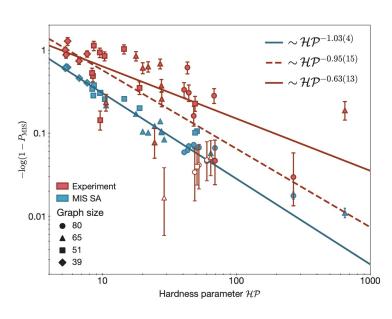
Challenge: Performance suppressed exponentially in hardness parameter.



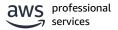
## Preliminary summary – part I





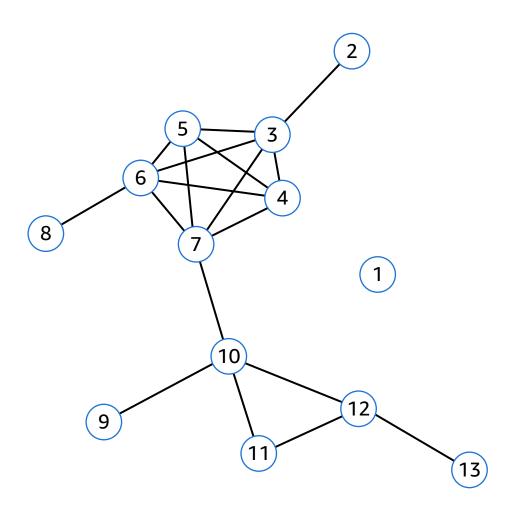


- Rydberg atom arrays provide native solver for the maximum independent set (MIS) problem on unit-disk (UD) graphs (MIS-UD problem).
- Performance suppressed exponentially in hardness parameter, for broad set of (classical and quantum) MCMC solvers.



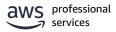
# Reduction techniques for the MIS problem

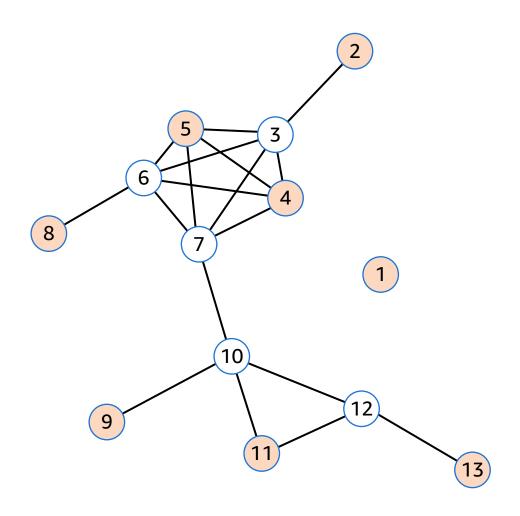




#### Basic property of the MIS problem:

Any clique can host at most one excitation (selected node).



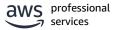


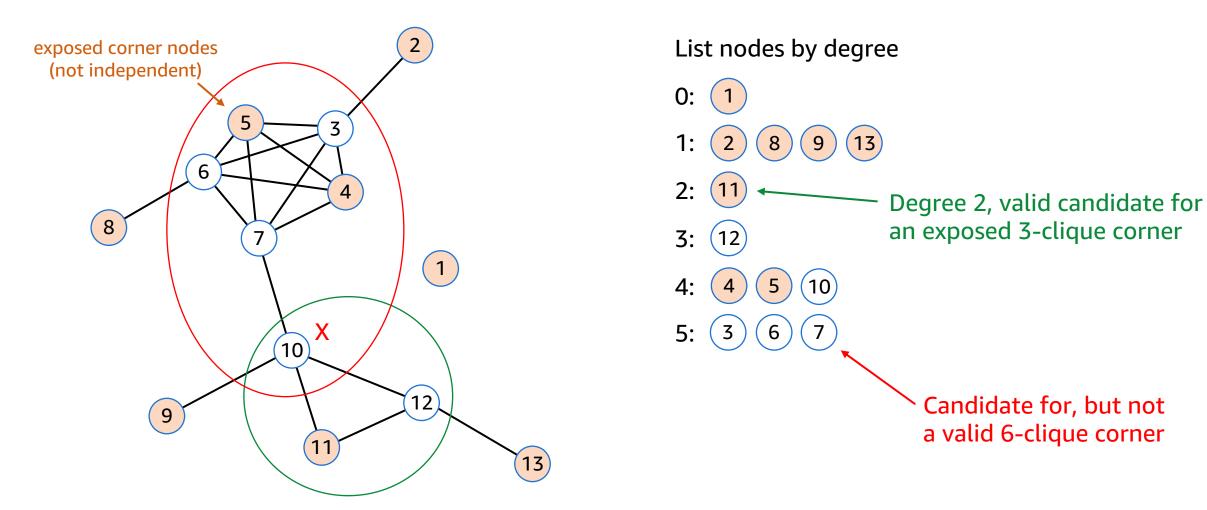
#### **Basic property of the MIS problem:**

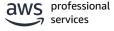
Any clique can host at most one excitation (selected node).

#### **Key idea:**

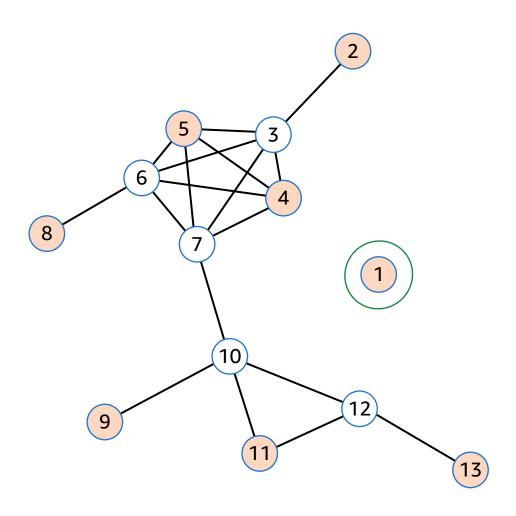
Reduce problem by removing exposed corner nodes.











#### List nodes by degree

0: (1

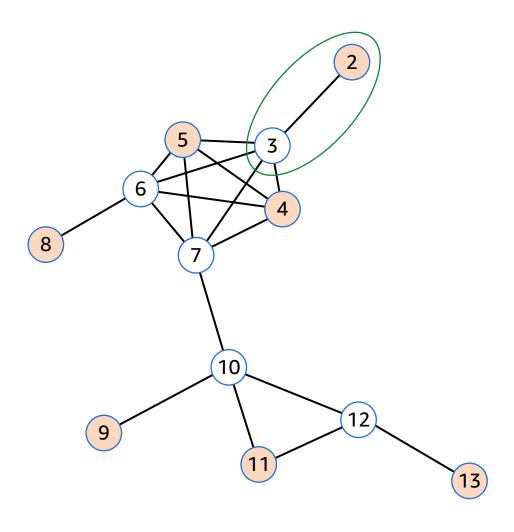
1: (2) (8) (9) (13

2: (11

3: (12

4: (4) (5) (10)

5: (3) (6) (7)



#### List nodes by degree

0:

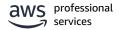
1: (2) (8) (9) (13

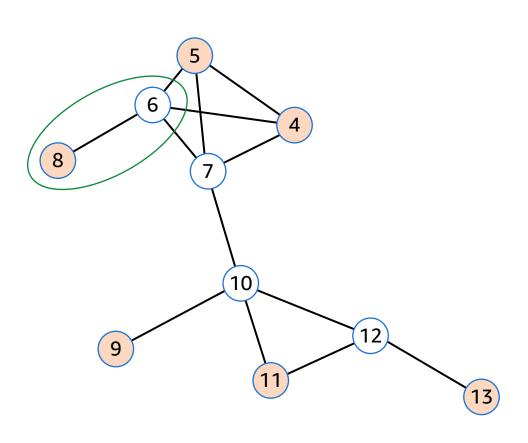
2: (11)

3: (12)

4: (4) (5) (10)

5: (3) (6) (7)





#### List nodes by degree

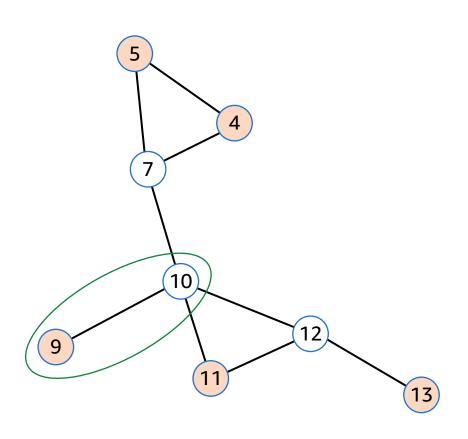
0:

1: (8) (9) (13)

2: (11

3: (4) (5) (12)

4: (6) (7) (10



List nodes by degree

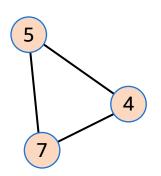
0:

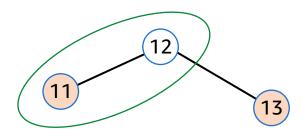
1: (9) (13

2: (5) (11

3: (4) (7) (12)

4: (10)





#### List nodes by degree

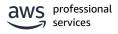
0:

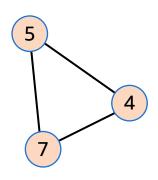
1: (11) (13

2: (4) (5) (7) (12

3:

4:





List nodes by degree

0: (13

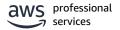
1:

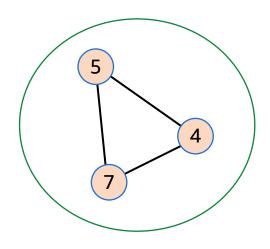
2: (4) (5) (7

3

4:







List nodes by degree

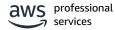
0:

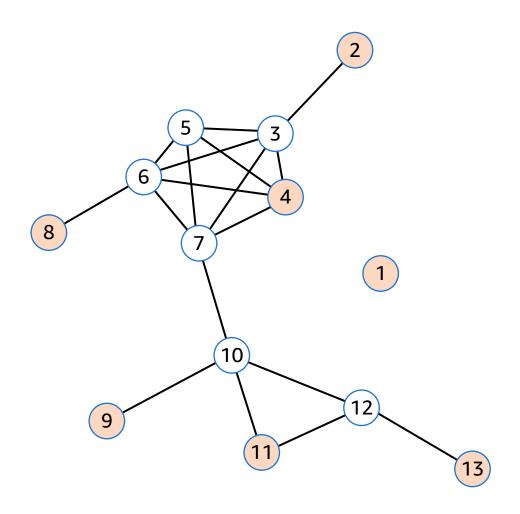
1:

2: (4) (5) (7)

3:

4:





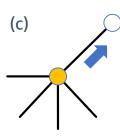
#### **Observations:**

- Graph fully reducible, with reduction factor  $\xi = 100\%$ .
- Reduction factor  $\xi \in [0,1]$  for input graphs with N nodes and output with n nodes:

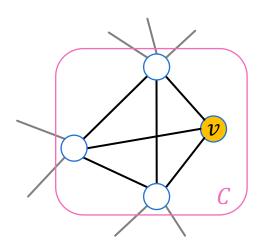
$$\xi = \frac{N - n}{N}$$

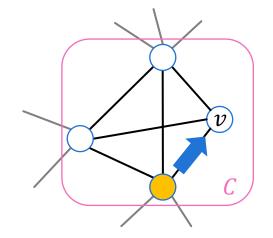
 Optimal MIS solution found by mere (clique-based) reduction.

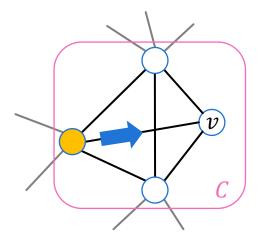


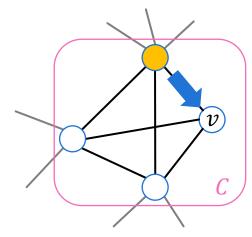


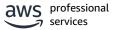
- Since v has no neighbors outside of the clique, by a cut-and-paste argument, it must be in some maximum independent set.
- Therefore, we can add v to the maximum independent set we are computing, and remove v and C from the graph.



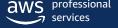






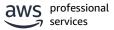


# Reduction experiments

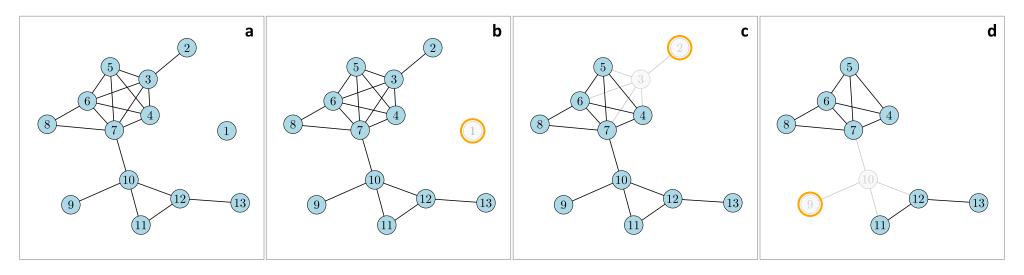


#### Reduction for real-world networks

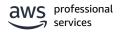
network	input graph ${\cal G}$			kernel $\mathcal K$ of input graph $\mathcal G$				run time	reduction
	nodes	edges	average degree	nodes	edges	components	largest component	[ms]	ξ
Florentine	15	20	2.67	0	0			0.38	100%
Zachary Karate	34	78	4.59	4	4	1	4	0.97	88.2%
Dolphins	62	159	5.13	20	30	<b>2</b>	12	1.51	67.7%
Les Miserables	77	254	6.60	0	0		_	2.21	100%
Jazz	198	2742	27.70	83	580	1	83	17.74	58.1%
C. Elegans	438	1519	6.94	19	26	1	19	14.32	95.7%
Email	1133	5451	9.62	315	818	1	315	44.05	72.2%
Cora	2708	5278	3.90	79	94	16	9	116.93	97.1%
Citeseer	3264	4536	2.78	217	341	26	83	67.30	93.4%
PubMed	19714	44281	4.49	16	23	2	11	665.06	99.9%



#### Preliminary summary – part II



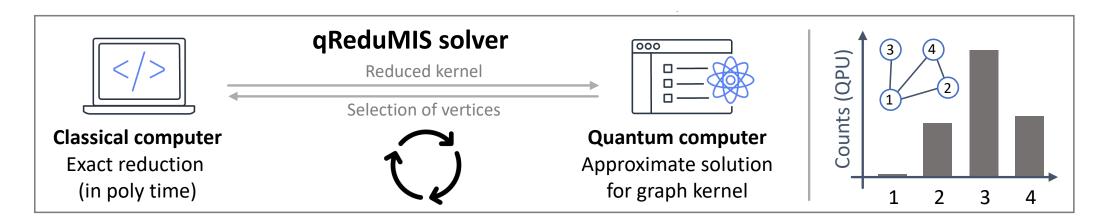
- Classical reduction (kernelization) is a powerful tool for MIS/MWIS problems.
- Reduction methods are provably optimal.
- Reduction methods are fast (e.g., linear-time for sparse graphs).
- Reduction methods insensitive to the hardness parameter.



## qReduMIS algorithm



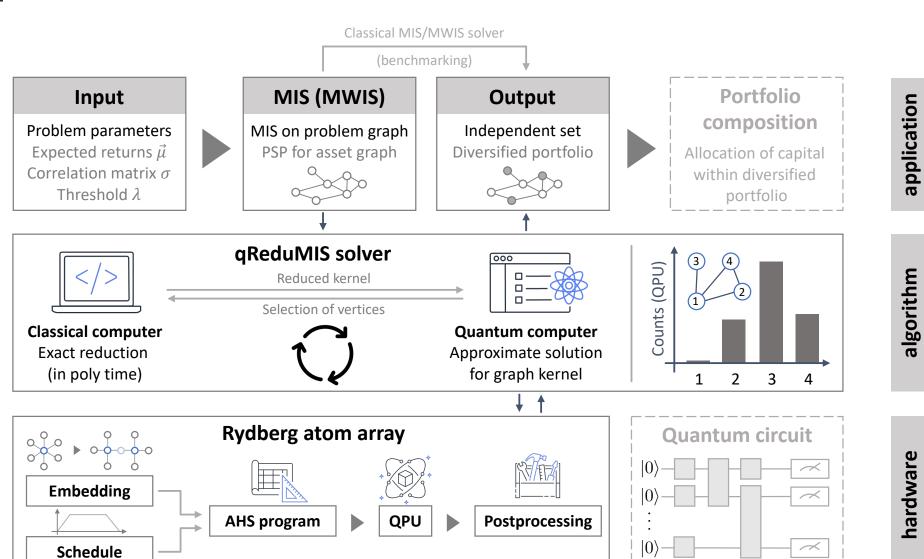
#### Motivation for qReduMIS



- Exploit (exact and fast) reduction as much as possible, akin to classical SOTA heuristics.
- Reuse reduction to boost performance at the expense of (small) overhead.
- Periodically leverage QPU's native sampling capabilities (in the form of sample persistence) to unblock reduction whenever necessary.



#### The qReduMIS framework



#### qReduMIS - High-level (pseudo-code) structure

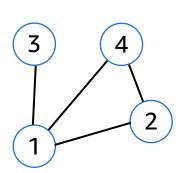
```
Input graph G = (V, E), RCL size K_{CL}, selection parameter \lambda # default K_{CL} = \lambda = 1
Global W = \emptyset, S = \emptyset, R = \emptyset # best solution, set of selected nodes, set of removed nodes
Procedure qReduMIS(G)
          if G empty return
          (K, s, r) \leftarrow \text{ClassicalReduce}(G) \# \text{ get kernel, selected and removed nodes}
          Append s to S, append r to R
          if K is empty then update and return W
          \{I_n\} \leftarrow \text{QuantumMIS}(K) # get intermediate independent sizes for kernel (incl. postprocessing) for
                                      # all n=1 ..., N_{shots} shots
          if |S| + \max\{|I_n|\} > |W| then update W # update incumbent
          Q, S, R \leftarrow \text{Select}(\{I_n\}, strategy = "in") \# \text{ select } \lambda \text{ frozen (fixed) degrees with "in"/"out" strategy}
                                                         # update sets S and R accordingly
          K' \leftarrow K[V_K \setminus Q] \# \text{ get inexact kernel}
          qReduMIS(K') # recurse on inexact kernel
```

**Return** *W* 



#### Quantum-informed identification of frozen nodes

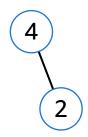
MIS={2, 3} MIS={3, 4}



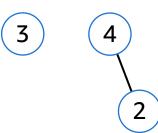
OUT 1 2 3 4

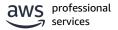
- Node 1 frozen
- Node 3 frozen

In-set strategy

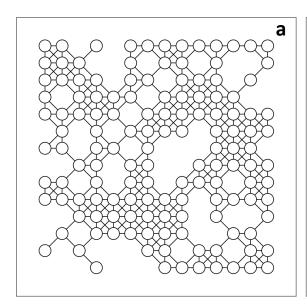


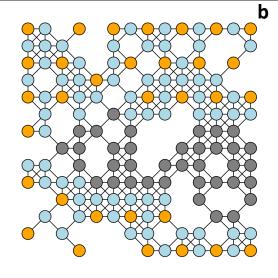
Out-of-set strategy

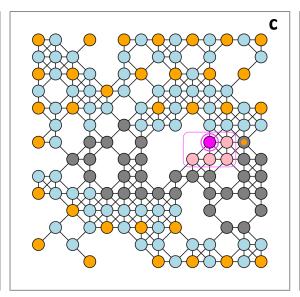


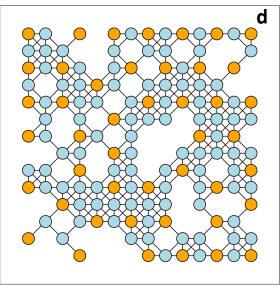


### Snapshots of the qReduMIS algorithm



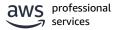




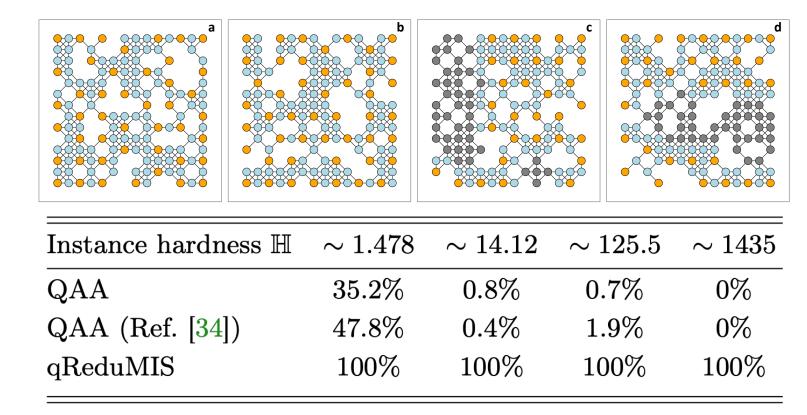


- Problem input: Sitediluted union-jack graph with 137 nodes.
- Hard instance with hardness ~1435.

- Classical reduction removes 100 nodes.
- Irreducible kernel (grey) found with 37 kernel nodes.
- QPU is called to unblock reduction.
- Node in pink identified as frozen node with high in-set probability.
- Remaining kernel fully reducible.
- MIS solution found with two classical reduction steps and one QPU call.

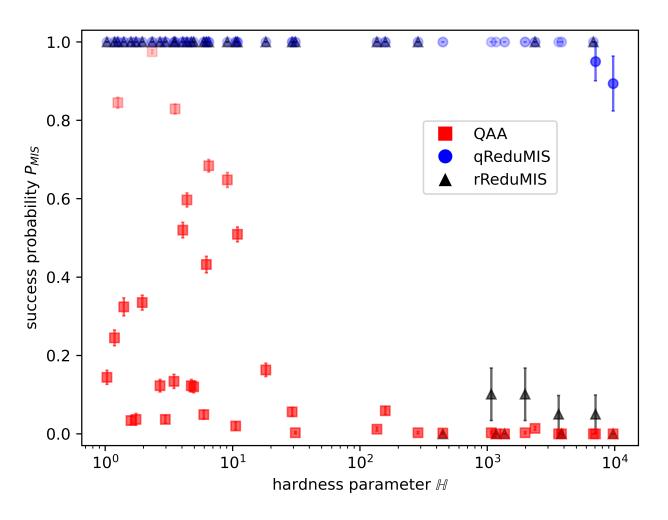


#### qReduMIS performance for example instances



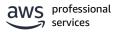
Average success probabilities  $P_{MIS}$  achieved with QAA and qReduMIS (with inset selection strategy) using the QuEra Aquila QPU for four test instances (all with n = 137 vertices).

### qReduMIS performance across larger testbed



#### **Systematic experiments:**

- Testbed with 39 random UJ instances with hardness ranging from ~ 1.03 up to ~ 9717.
- qReduMIS solves most instances to optimality with  $P_{MIS} = 1$  and maintains a nonzero success rate with average  $P_{MIS} \gtrsim 89\%$  throughout the testbed.



#### qReduMIS is hardware agnostic

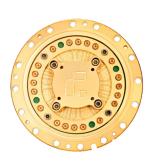
- The qReduMIS hardware layer can involve analog or digital quantum devices.
- Task: To unblock reduction, solve MIS problem on (kernel) graph:

$$H(\mathbf{n}) = -\sum_{i \in \mathcal{V}_{\mathcal{K}}} n_i + U \sum_{(i,j) \in \mathcal{E}_{\mathcal{K}}} n_i n_j, \qquad \hat{H}_{\text{cost}} = \sum_{i,j} J_{ij} \hat{Z}_i \hat{Z}_j + \sum_i h_i \hat{Z}_i,$$

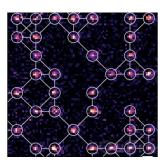
• **Approach**: Prepare low-energy state via optimized control parameters  $\theta$ :

$$|\Psi(\theta)\rangle = \mathcal{U}(\theta)|\Psi_{\mathrm{initial}}\rangle,$$

- Gate-based QAOA (SC qubits, trapped ions, ...).
- Ising machines, including SC annealers (D-Wave).
- AHS with Rydberg annealers (neutral atoms).
- Impact beyond Rydberg community...





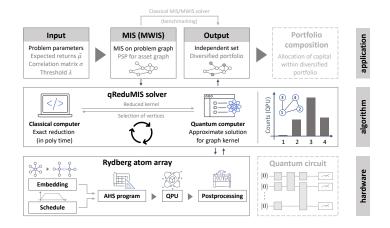


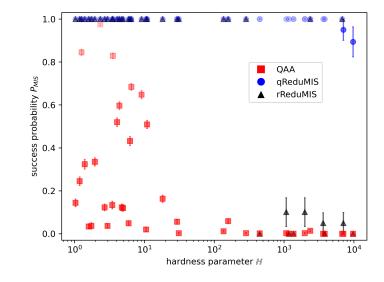
# Summary and outlook

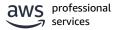


#### **Summary and outlook**

- Novel quantum-informed qReduMIS algorithm.
- Kernelization is applied in tandem with a quantum coprocessor that helps guide the repeated reduction process through the identification of frozen variables.
- Successful experiments with up to 231 qubits on QuEra, showing that qReduMIS can tackle fundamental performance limitations.
- Outlook: Future experiments based on alternative hardware platforms (such as superconducting qubits or trapped ions).







## Thank you.

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