The Search for Applications of Quantum Computing in Industry

qReduMIS: A Quantum-Informed Reduction Algorithm for the Maximum Independent Set Problem

Martin Schuetz
Global Practice Lead
Amazon Advanced Solutions Lab

Agenda

- Introduction
- Overview: Challenge, Situation, Solution
- The MIS problem and Rydberg atom arrays
- Deep dive into qReduMIS algorithm
- Results: Performance analysis
- Summary and outlook

Quantum computing at AWS

Amazon Braket

Democratize quantum computing

Fully managed service that makes it easy to explore and experiment

AWS Center for Quantum Computing

Push the boundaries

Research quantum algorithms, hardware, and networking

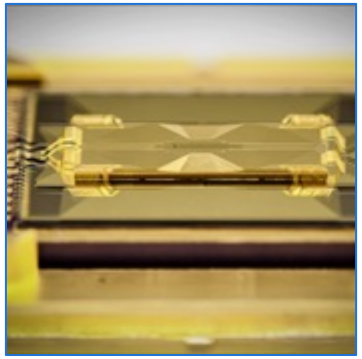
Amazon Advanced Solutions Lab

Provide expert guidance

Practical and cross-discipline support and collaboration

Quantum devices available on Amazon Braket

IONQ

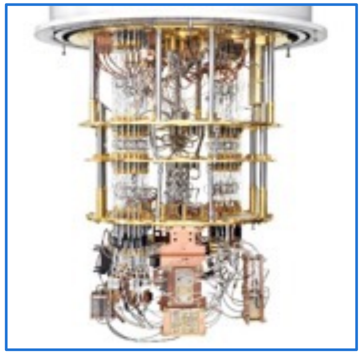


Ion-trap QPUs

Aria (25 qubits)

Forte (36 qubits)

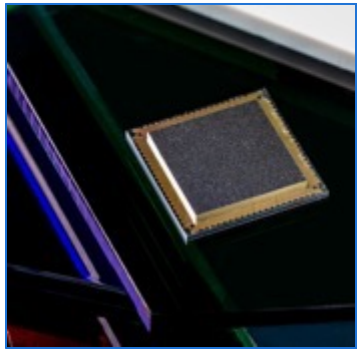
rigetti



Superconducting QPU

Ankaa-3 (84 qubits)

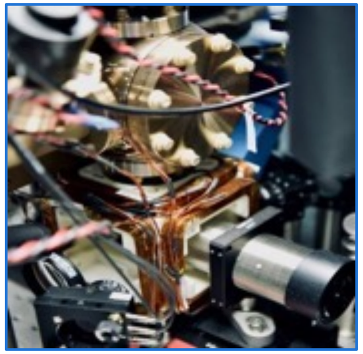
IQM



Superconducting QPU

Garnet (20 qubits)

QuEra



Analog neutral atom QPU

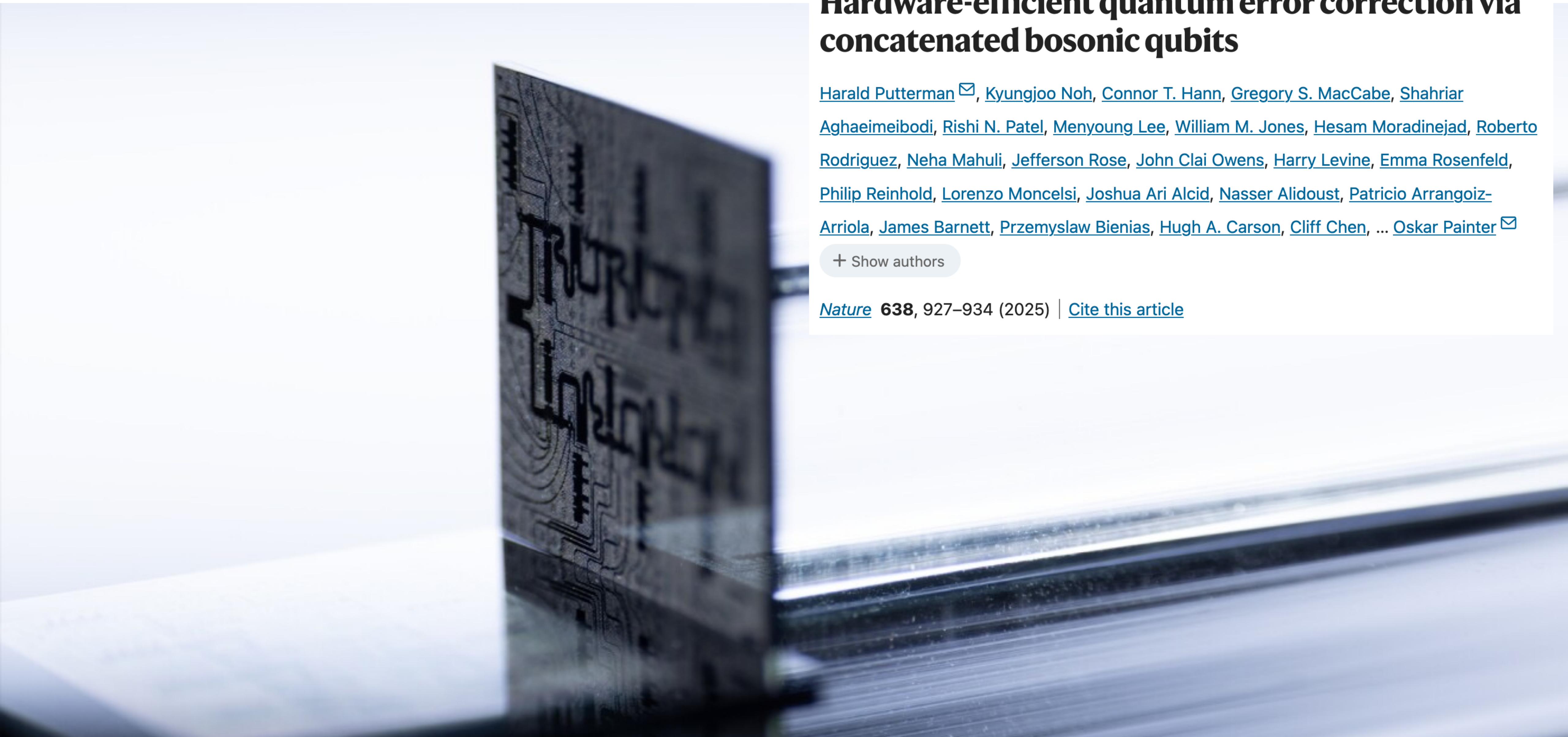
Aquila (256 qubits)

Quantum circuit

Analog program

AWS Center for Quantum Computing

Ocelot



Article | Open access | Published: 26 February 2025

Hardware-efficient quantum error correction via concatenated bosonic qubits

Harald Putterman, Kyungjoo Noh, Connor T. Hann, Gregory S. MacCabe, Shahriar Aghaeimeibodi, Rishi N. Patel, Menyoung Lee, William M. Jones, Hesam Moradinejad, Roberto Rodriguez, Neha Mahuli, Jefferson Rose, John Clai Owens, Harry Levine, Emma Rosenfeld, Philip Reinhold, Lorenzo Moncelsi, Joshua Ari Alcid, Nasser Alidoust, Patricio Arrangoiz-Arriola, James Barnett, Przemyslaw Bienias, Hugh A. Carson, Cliff Chen, ... Oskar Painter

+ Show authors

Nature **638**, 927–934 (2025) | Cite this article

Amazon Advanced Solutions Lab (ASL)

What we offer

Deep expertise in advanced technologies to design and *build* cutting-edge solutions for challenging business problems at scale. Technologies leveraged: operations research, mathematical optimization, quantum and high-performance computing, machine learning, computational chemistry, and combinations thereof.

• • •

Machine Learning

*Use state-of-the-art machine learning for **predictive** analytics and make better predictions about the future.*

Optimization

*Leverage optimization for **prescriptive** analytics and empower customers to make the best possible business decisions.*

Quantum Technologies

Help customers get ready for quantum technologies, through hands-on research engagements and C-Suite briefings.

• • •

Example use cases: Vehicle routing • Flight scheduling • Sports scheduling • Workforce rostering • Ad placement • Clinical trial enrollment • Manufacturing process optimization • Forecasting • Portfolio optimization • Reinsurance optimization • Middle-mile network optimization • Unit commitment in energy applications, …

Our customers & partners prepare for the quantum future

Optimization of Robot Trajectory Planning with Nature-Inspired and Hybrid Quantum Algorithms

Martin J. A. Schuetz,[1,2,3] J. Kyle Brubaker,[2] Henry Montagu,[1,2,3] Yannick van Dijk,[4] Johannes Klepsch,[4] Philipp Ross,[4] Andre Luckow,[4] Mauricio G. C. Resende,[5,6] and Helmut G. Katzgraber[1,2,3,6]

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[5] *Amazon.com, Inc., Bellevue, Washington 98004, USA*
[6] *University of Washington, Seattle, Washington 98195, USA*

(Dated: June 9, 2022)

arXiv:2206.03651

Explainable AI using expressive Boolean formulas

Gili Rosenberg,[1] J. Kyle Brubaker,[1] Martin J. A. Schuetz,[1,2] Grant Salton,[1,2,3] Zhihuai Zhu,[1] Elton Yechao Zhu,[4] Serdar Kadıoğlu,[5] Sima E. Borujeni,[4] and Helmut G. Katzgraber[1]

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[3] *California Institute of Technology, Pasadena, CA 91125, USA*
[4] *Fidelity Center for Applied Technology, FMR LLC, Boston, MA 02210, USA*
[5] *AI Center of Excellence, FMR LLC, Boston, MA 02210, USA*

(Dated: June 8, 2023)

arXiv:2306.03976

End-to-end resource analysis for quantum interior point methods and portfolio optimization

Alexander M. Dalzell,[1,2] B. David Clader,[3] Grant Salton,[4,1,2] Mario Berta,[1,2,5,6] Cedric Yen-Yu Lin,[7] David A. Bader,[3,8] Nikitas Stamatopoulos,[3] Martin J. A. Schuetz,[4,1] Fernando G.S.L. Brandão,[1,2] Helmut G. Katzgraber,[4,1,9] and William J. Zeng[3]

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arXiv:2211.12489

Hardness of the Maximum Independent Set Problem on Unit-Disk Graphs and Prospects for Quantum Speedups

Ruben S. Andrist,[1,*] Martin J. A. Schuetz,[1,2,*] Pierre Minssen,[3,*] Romina Yalovetzky,[3,*] Shouvanik Chakrabarti,[3] Dylan Herman,[3] Niraj Kumar,[3] Grant Salton,[1,2,4] Ruslan Shaydulin,[3] Yue Sun,[3] Marco Pistoia,[3,†] and Helmut G. Katzgraber[1,†]

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(Dated: July 19, 2023)

arXiv:2307.09442

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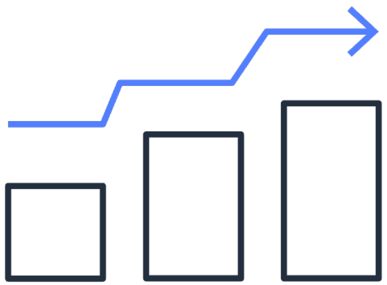
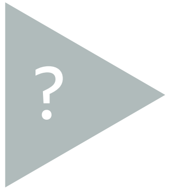
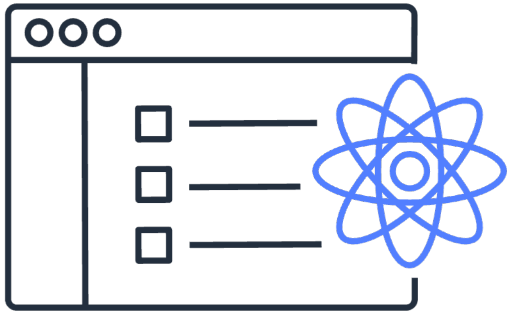
Overview: Challenge, Situation, Solution

The challenge







Real-world optimization problems

- Large scales (e.g., 1000+ assets)
- Constraints (e.g., budget constraints)
- Stringent time and quality requirements

Near-term quantum computers

- Small scales (hundreds of qubits)
- Unconstrained Hamiltonian encoding
- Hardware noise and other imperfections

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The situation around 2022

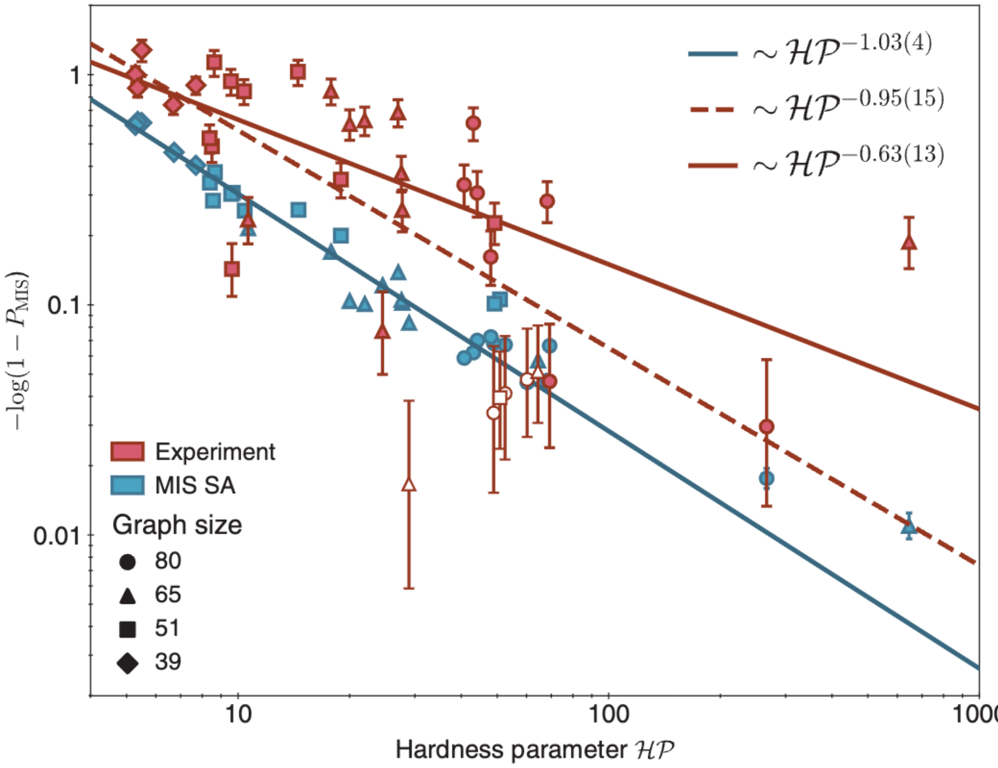
Quantum optimization of maximum independent set using Rydberg atom arrays

S. EBADI, A. KEESLING, M. CAIN, T. T. WANG, H. LEVINE, D. BLUVSTEIN, G. SEMEGHINI, A. OMRAN, J.-G. LIU, R. SAMAJDAR, X.-Z. LUO, B. NASH, X. GAO, B. BARAK, E. FARHI, S. SACHDEV, N. GEMELKE, L. ZHOU, S. CHOI, H. PICHLER, S.-T. WANG, M. GREINER, V. VULETIĆ, AND M. D. LUKIN fewer Authors Info & Affiliations

Science

SCIENCE • 5 May 2022 • Vol 376, Issue 6598 • pp. 1209-1215 • DOI: 10.1126/science.abo6587

Realizing quantum speedup for practically relevant, computationally hard problems is a central challenge in quantum information science. Using Rydberg atom arrays with up to 289 qubits in two spatial dimensions, we experimentally investigate quantum algorithms for solving the maximum independent set problem. We use a hardware-efficient encoding associated with Rydberg blockade, realize closed-loop optimization to test several variational algorithms, and subsequently apply them to systematically explore a class of graphs with programmable connectivity. We find that the problem hardness is controlled by the solution degeneracy and number of local minima, and we experimentally benchmark the quantum algorithm's performance against classical simulated annealing. On the hardest graphs, we observe a superlinear quantum speedup in finding exact solutions in the deep circuit regime and analyze its origins.



Our collaboration with JPMorgan Chase

PHYSICAL REVIEW RESEARCH 5, 043277 (2023)

Hardness of the maximum-independent-set problem on unit-disk graphs and prospects for quantum speedups

- Study the hardness of the MIS-UD problem
- Problem native for Rydberg quantum devices

PHYSICAL REVIEW RESEARCH 7, 023142 (2025)

Decomposition pipeline for large-scale portfolio optimization with applications to near-term quantum computing

- Decomposition pipeline for large portfolio optimization problems
- Reduction by ~80% for real-world instances

Quantum Compilation Toolkit for Rydberg Atom Arrays with Implications for Problem Hardness and Quantum Speedups

arXiv:2412.14976v2 [quant-ph] 14 Jan 2025

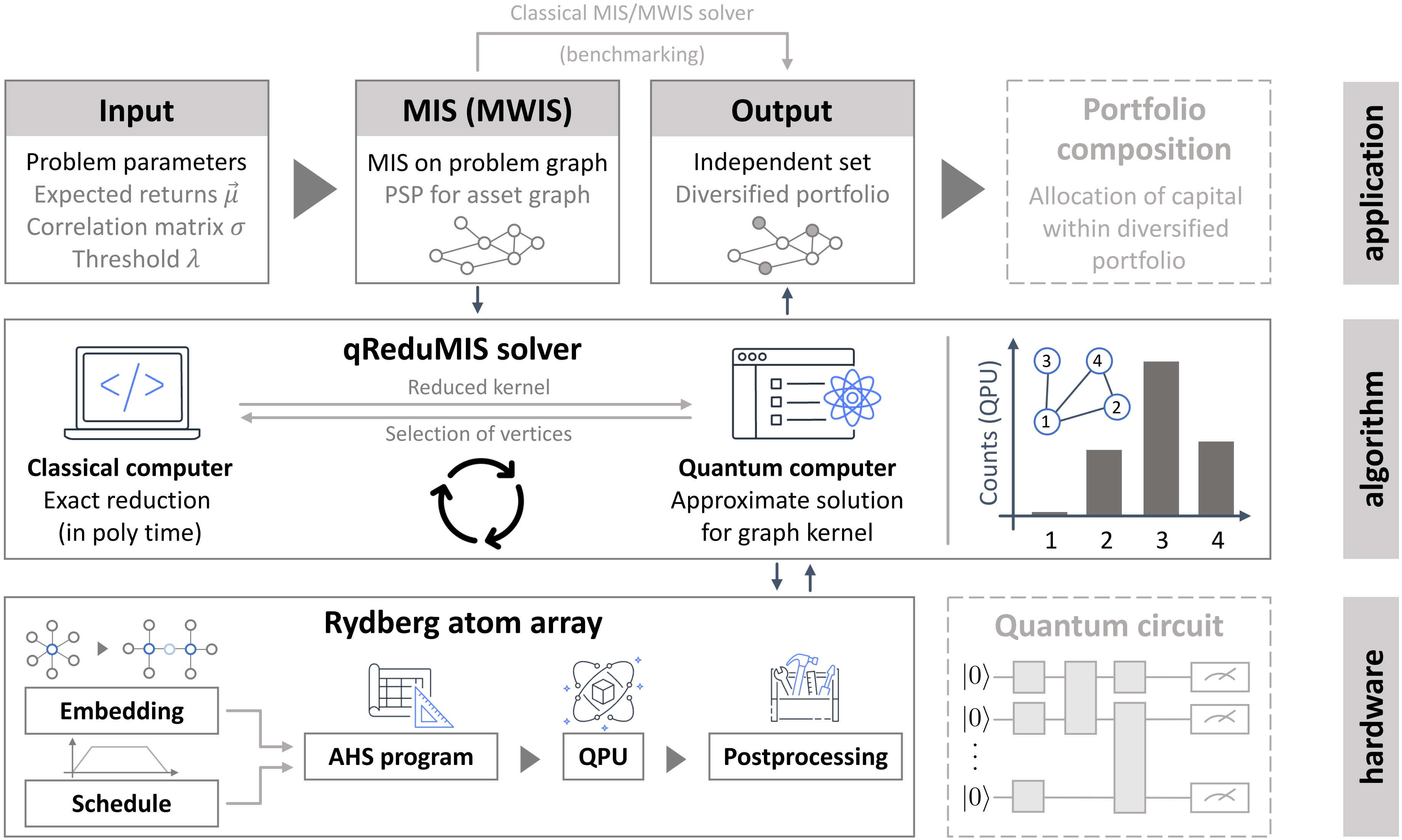
- Compilation toolkit for Rydberg atom arrays
- Hardware-efficient embedding with optimized overhead

qReduMIS: A Quantum-Informed Reduction Algorithm for the Maximum Independent Set Problem

arXiv:2503.12551v1 [quant-ph] 16 Mar 2025

- Quantum-informed qReduMIS algorithm
- Integration of exact classical reduction with QPU as co-processor

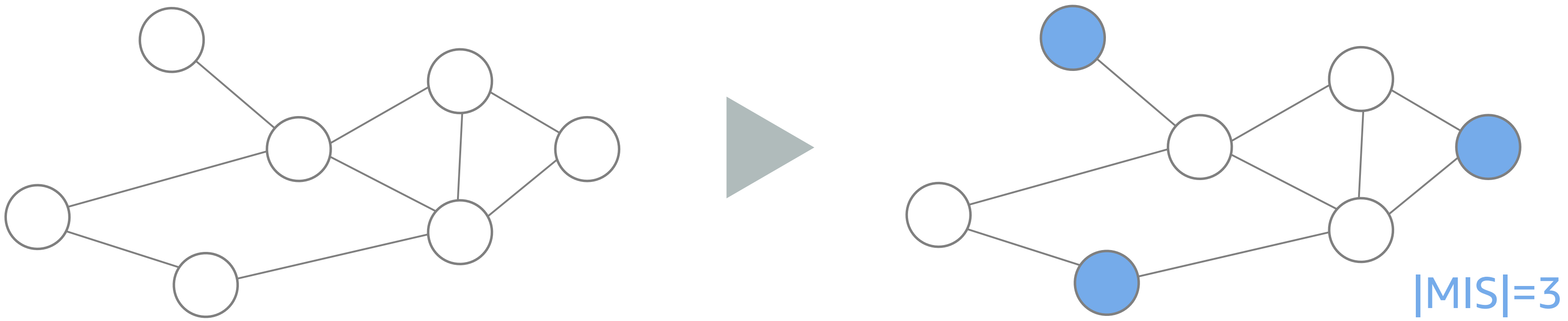
The solution: qReduMIS algorithm



The MIS problem and Rydberg atom arrays

The MIS problem

- **Problem statement**: Given an undirected graph, find largest independent set of vertices (i.e., the largest subset of vertices such that no edges connect any pair in the set).



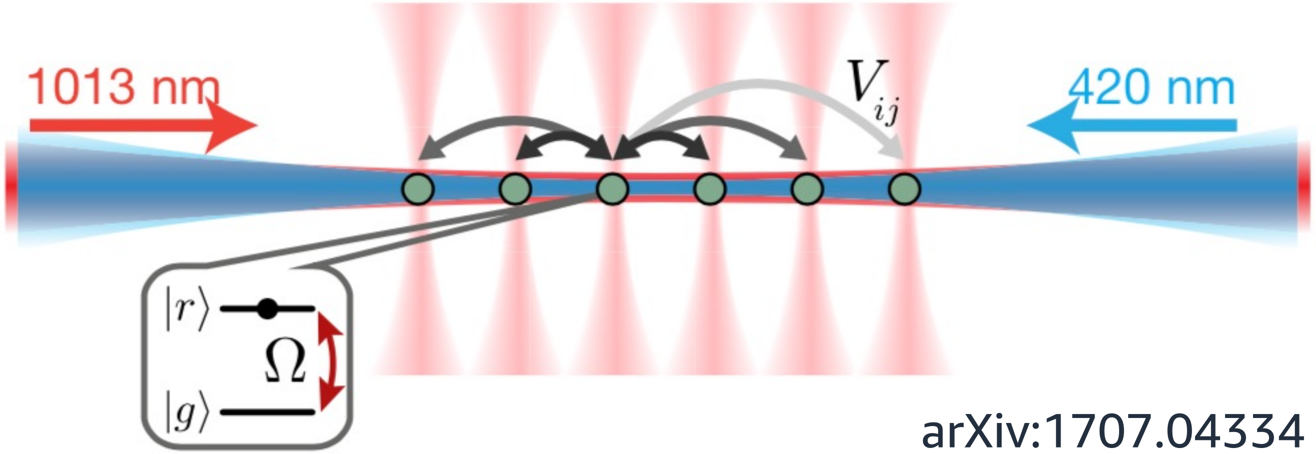
- MIS problem dual to maximum clique and minimum vertex cover problems.
- **Practical applications**: Map labeling, network design, vehicle routing, quantitative finance, …

Portfolio selection

- **Problem statement**: Given a (large) universe of assets, find the largest set of uncorrelated assets.

- **Solution**: Invest in 3 uncorrelated assets {A, C, G}, out of 7 assets.

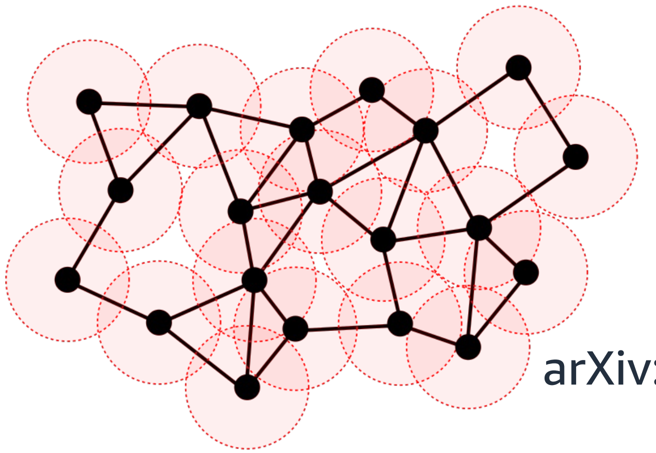
Rydberg atom arrays in a nutshell



arXiv:1707.04334

Effective spin Hamiltonian:

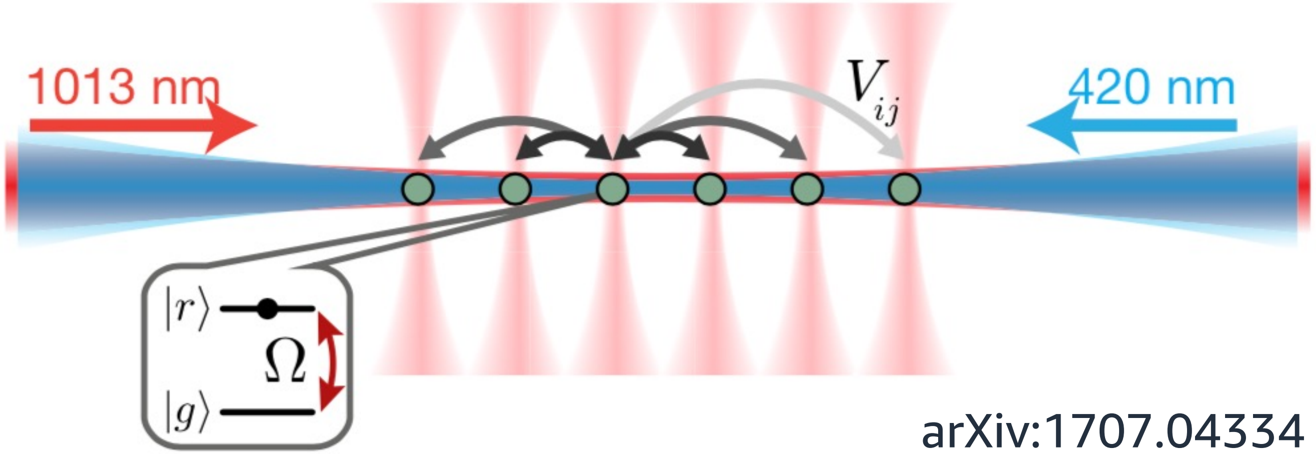
$$H = \sum_i \Omega \sigma_i^x - \Delta \sigma_i^z + \sum_{i<j} V_{ij} n_i n_j$$



arXiv:2205.08500

- Customizable geometry of Rydberg atoms via trapping potentials
- Strong Rydberg interactions lead to a unit disk (UD) graph connectivity
- Dynamical control of the Hamiltonian parameters

Rydberg atom arrays in a nutshell

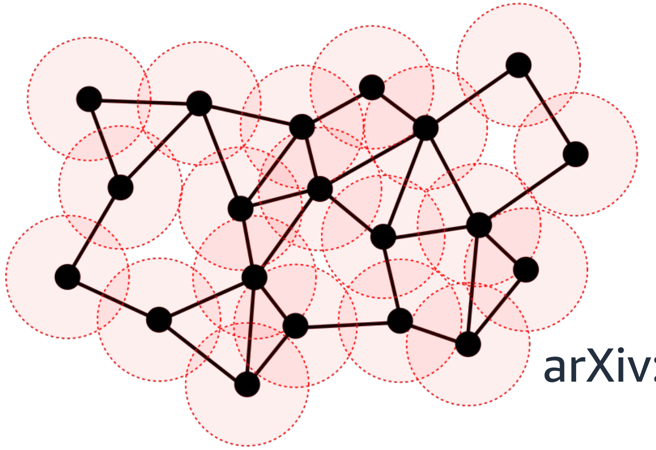


arXiv:1707.04334

Quantum optimization:

$$H = \sum_i \Omega \sigma_i^x - \Delta \sigma_i^z + \sum_{i<j} V_{ij} n_i n_j$$

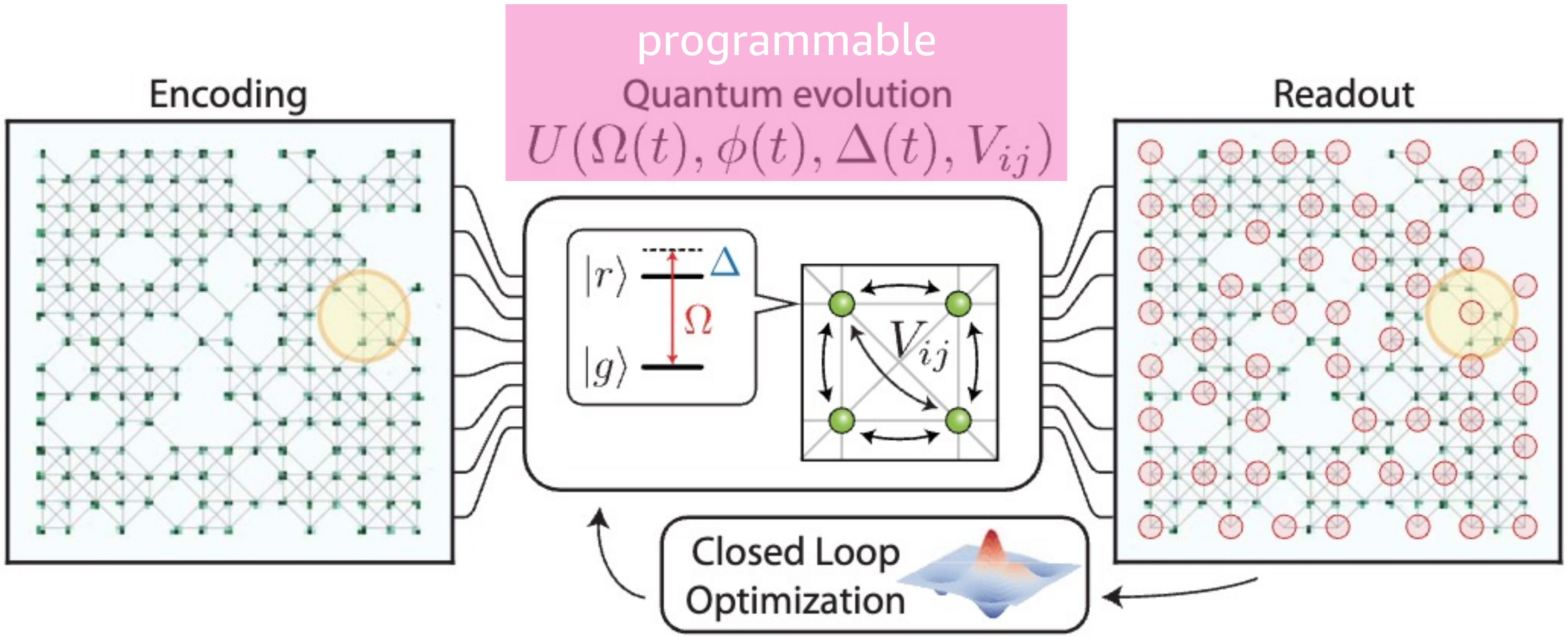
driver — MIS cost

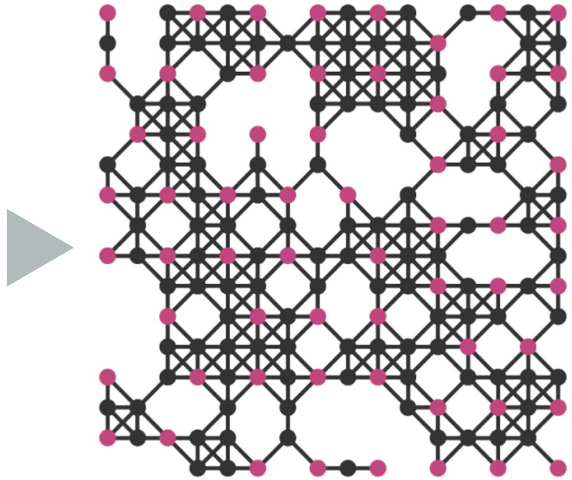


arXiv:2205.08500

- Customizable geometry of Rydberg atoms via trapping potentials
- Strong Rydberg interactions lead to a unit disk (UD) graph connectivity
- Dynamical control of the Hamiltonian parameters

Quantum optimization for MIS on random UJ graphs

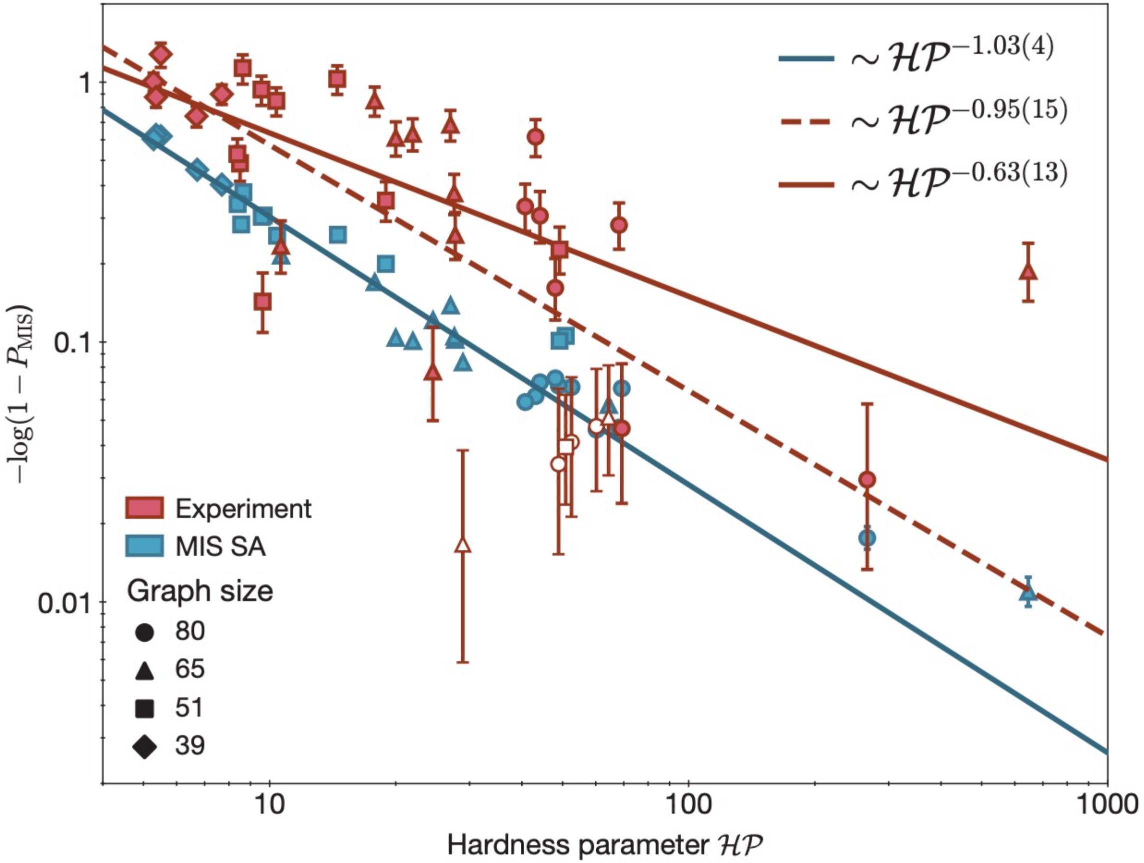




MIS on random union-jack (UJ) graph

Ebadi *et al.*, arXiv:2202.09372 [Science **376**, 1209 (2022)]

Performance on random UJ graphs



Ebadi *et al.*, arXiv:2202.09372 [Science 376, 1209 (2022)]

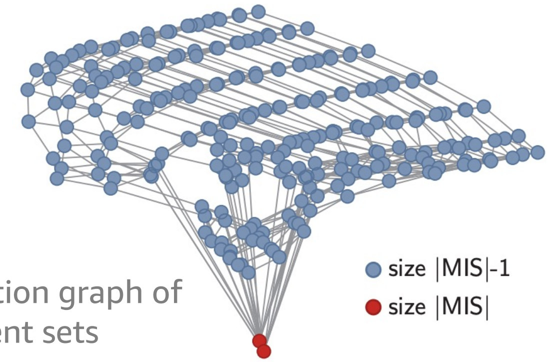
- Quantum algorithm reported to outperform classical simulated annealing (SA).
- Performance scaling:

$$P_{\mathrm{MIS}} \sim 1 - \exp\left(-C\mathbb{H}^{-1.03}\right) \quad \text{SA}$$

$$P_{\mathrm{MIS}} \sim 1 - \exp\left(-C\mathbb{H}^{-0.63}\right) \quad \text{QA}$$

- Hardness parameter:

$$\mathbb{H} = \frac{D_{\mathrm{MIS}-1}}{|\mathrm{MIS}| \cdot D_{\mathrm{MIS}}}$$



Configuration graph of independent sets

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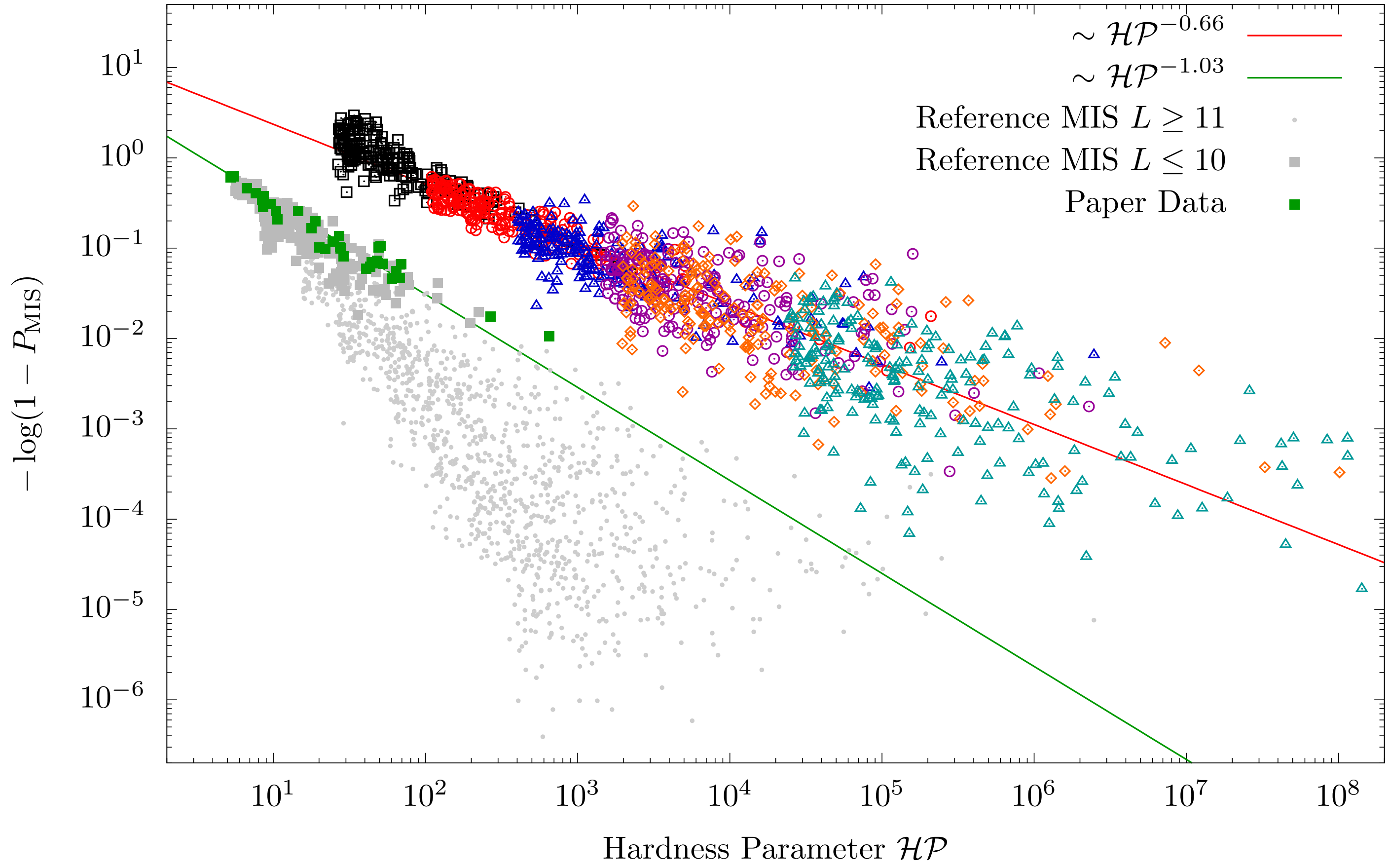
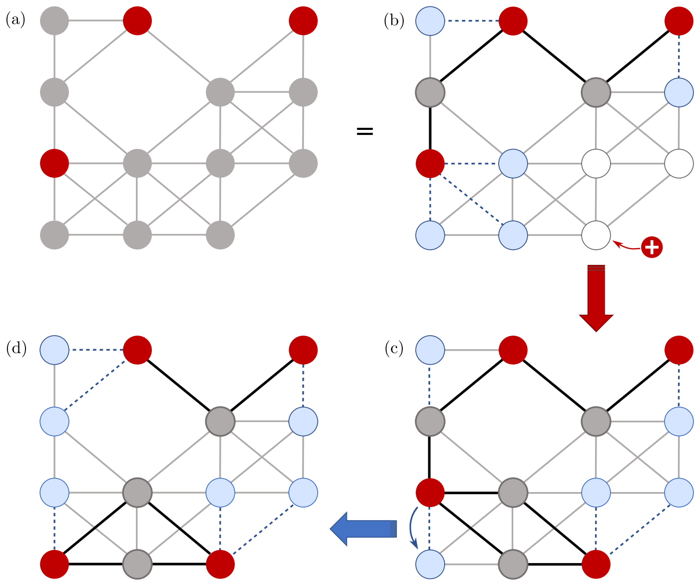
Results for native SA solver

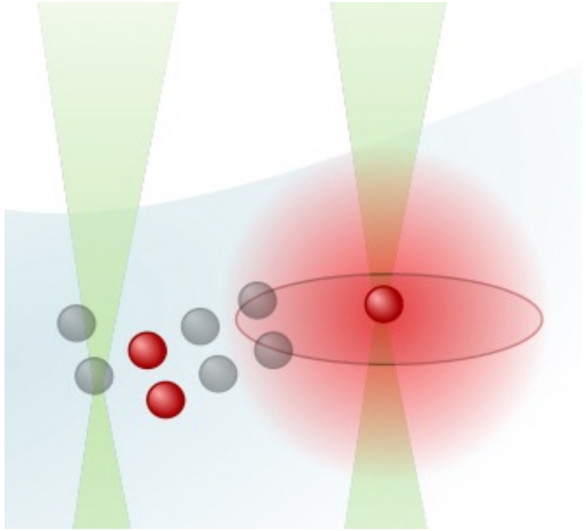


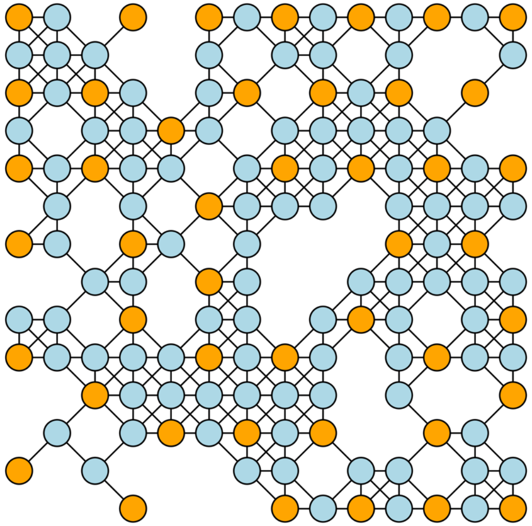


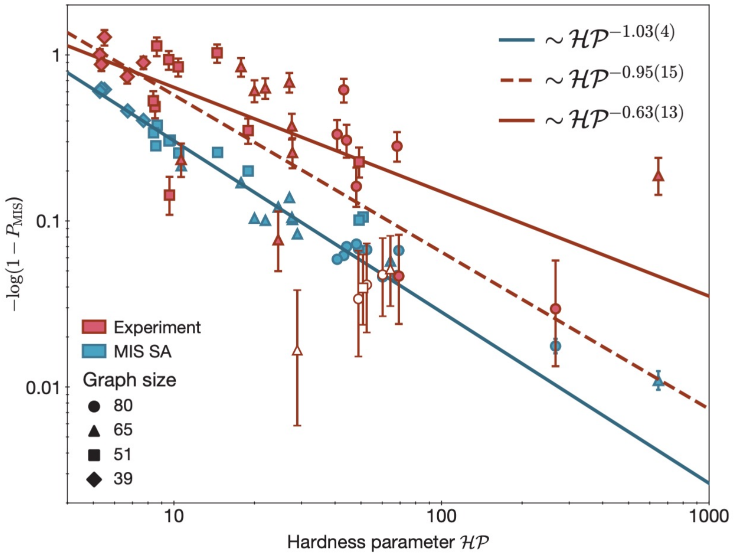
arXiv:2307.09442

Challenge: Performance suppressed exponentially in hardness parameter.

Preliminary summary – part I



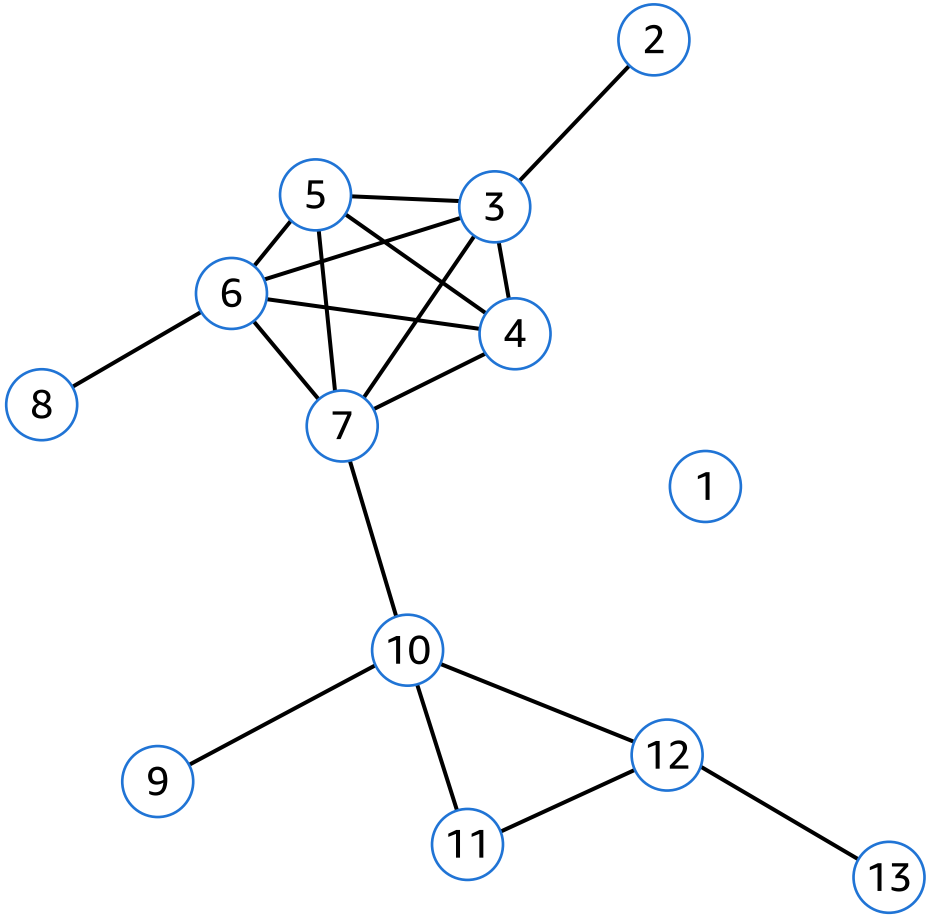




- Rydberg atom arrays provide native solver for the maximum independent set (MIS) problem on unit-disk (UD) graphs (MIS-UD problem).
- Performance suppressed exponentially in hardness parameter, for broad set of (classical and quantum) MCMC solvers.

Reduction techniques for the MIS problem

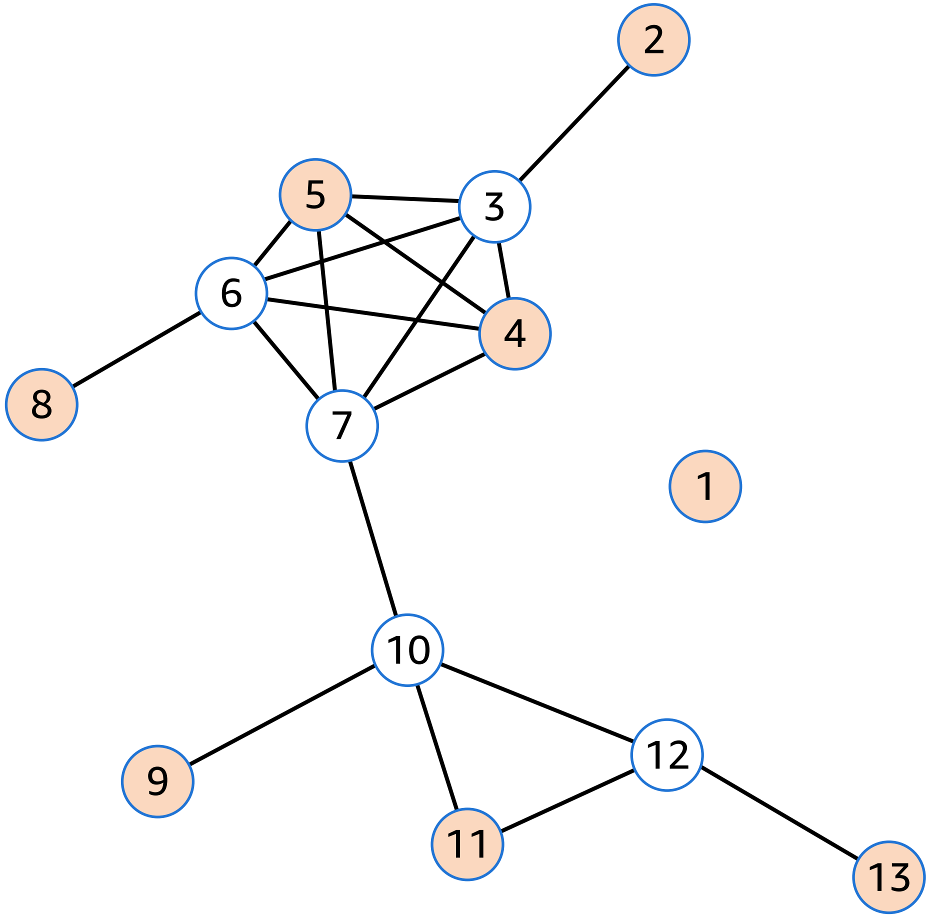
Reduction/kernelization for the MIS problem



Basic property of the MIS problem:

Any clique can host at most one excitation (selected node).

Reduction/kernelization for the MIS problem



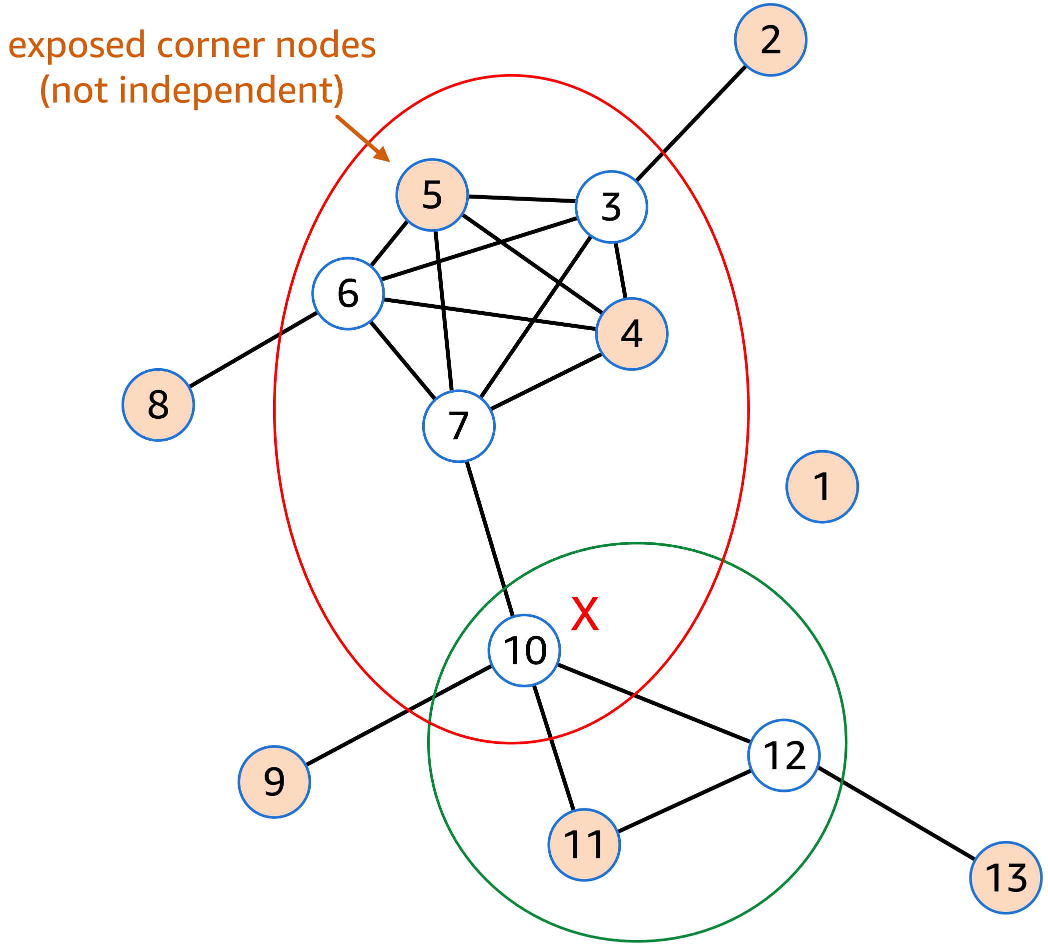
Basic property of the MIS problem:

Any clique can host at most one excitation (selected node).

Key idea:

Reduce problem by removing exposed corner nodes.

Reduction/kernelization for the MIS problem



List nodes by degree

0: 1

1: 2 8 9 13

2: 11

3: 12

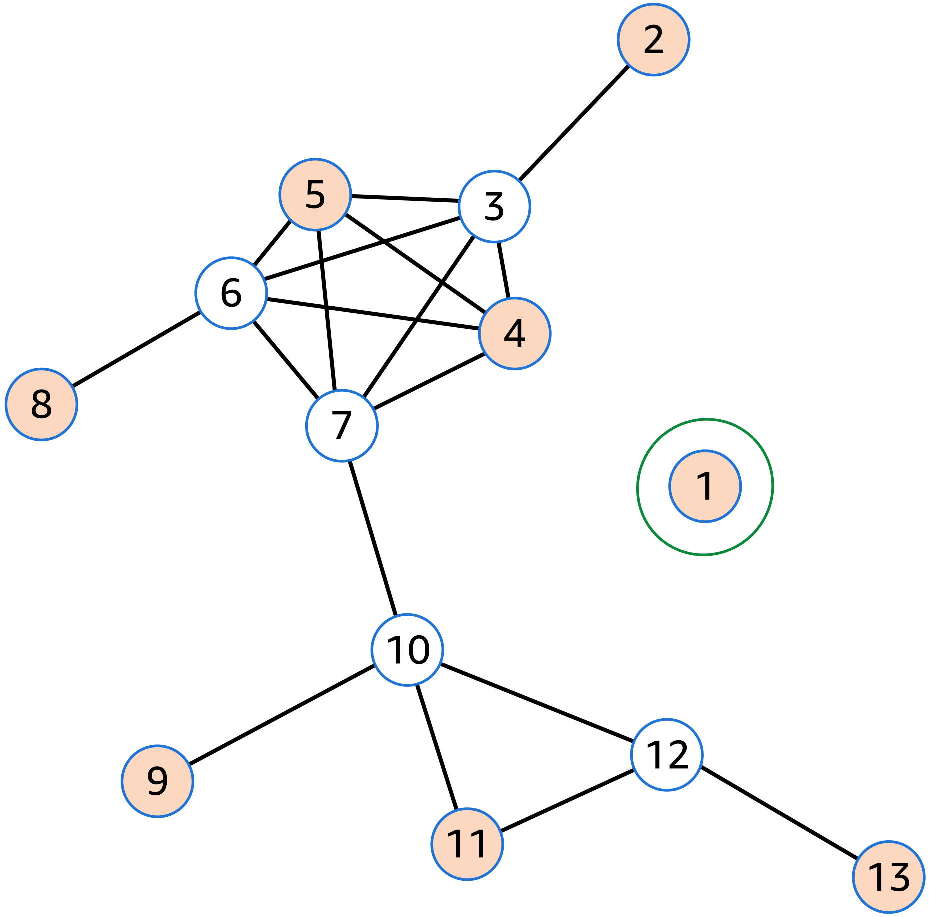
4: 4 5 10

5: 3 6 7

Degree 2, valid candidate for an exposed 3-clique corner

Candidate for, but not a valid 6-clique corner

Reduction/kernelization for the MIS problem



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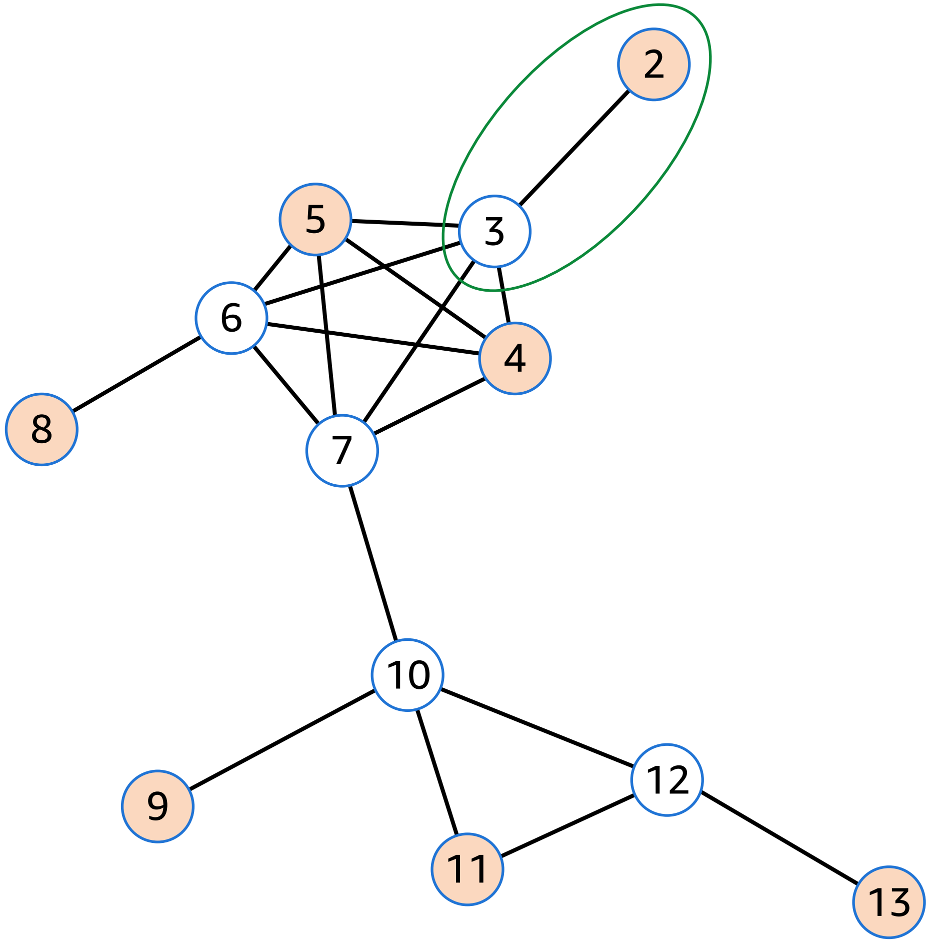
Reduction/kernelization for the MIS problem



List nodes by degree

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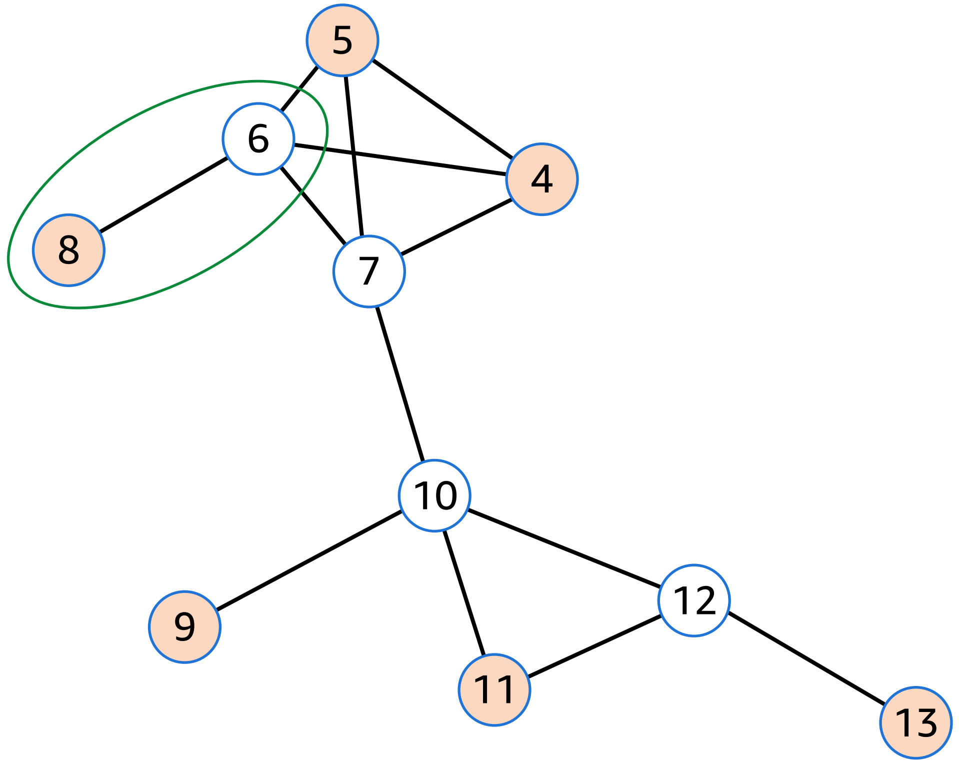
Reduction/kernelization for the MIS problem



List nodes by degree

0:

1: 8 9 13

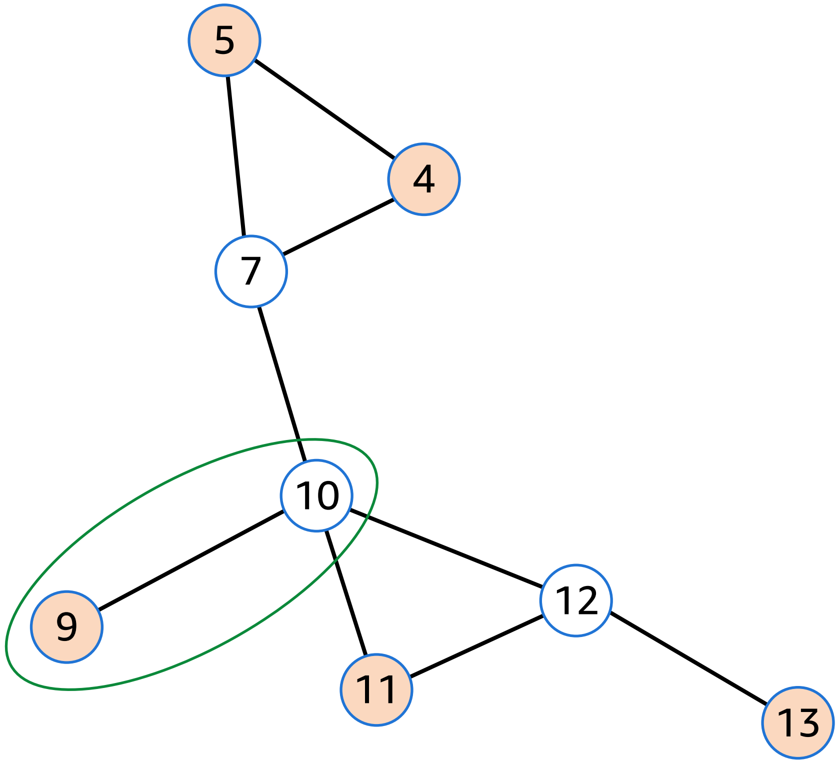
2: 11

3: 4 5 12

4: 6 7 10

5:

Reduction/kernelization for the MIS problem



List nodes by degree

0:

1: 9 13

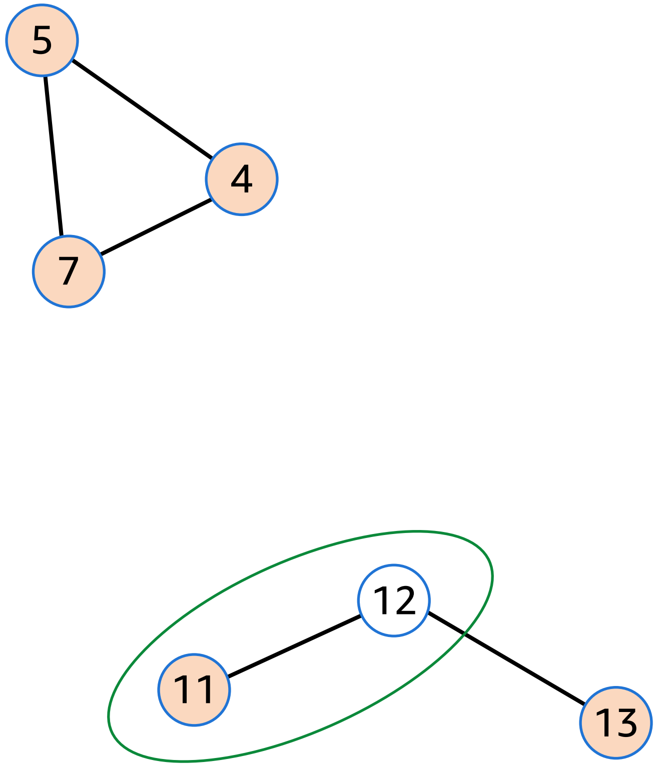
2: 5 11

3: 4 7 12

4: 10

5:

Reduction/kernelization for the MIS problem



List nodes by degree

0:

1: 11 13

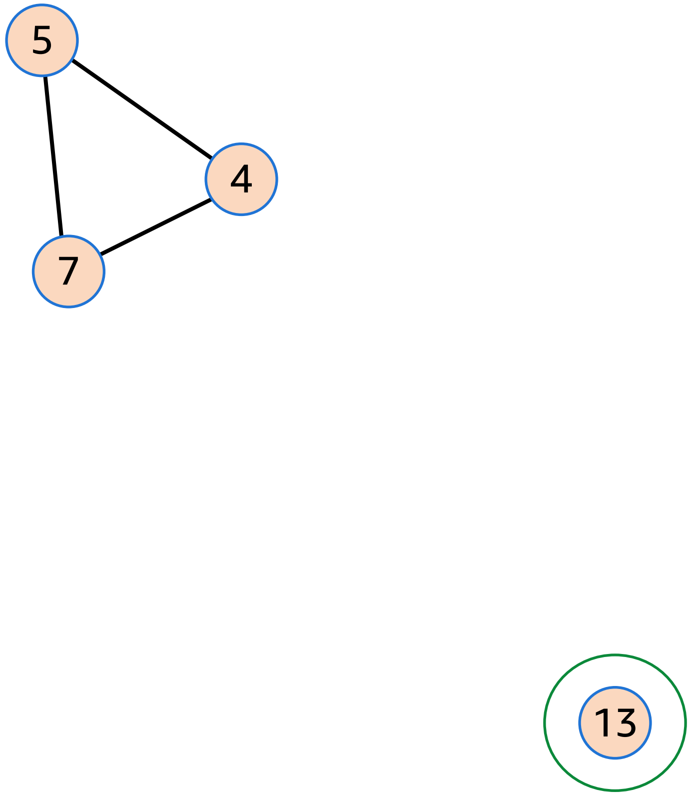
2: 4 5 7 12

3:

4:

5:

Reduction/kernelization for the MIS problem



List nodes by degree

0: 13

1:

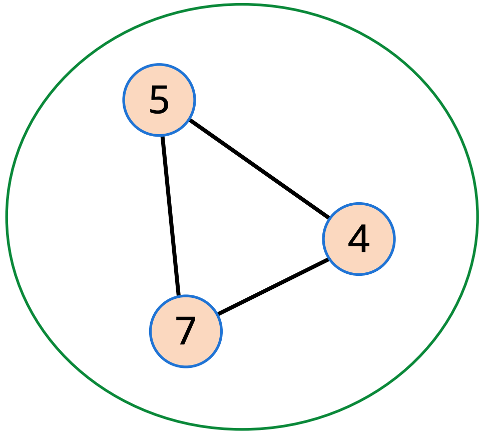
2: 4 5 7

3:

4:

5:

Reduction/kernelization for the MIS problem



List nodes by degree

0:

1:

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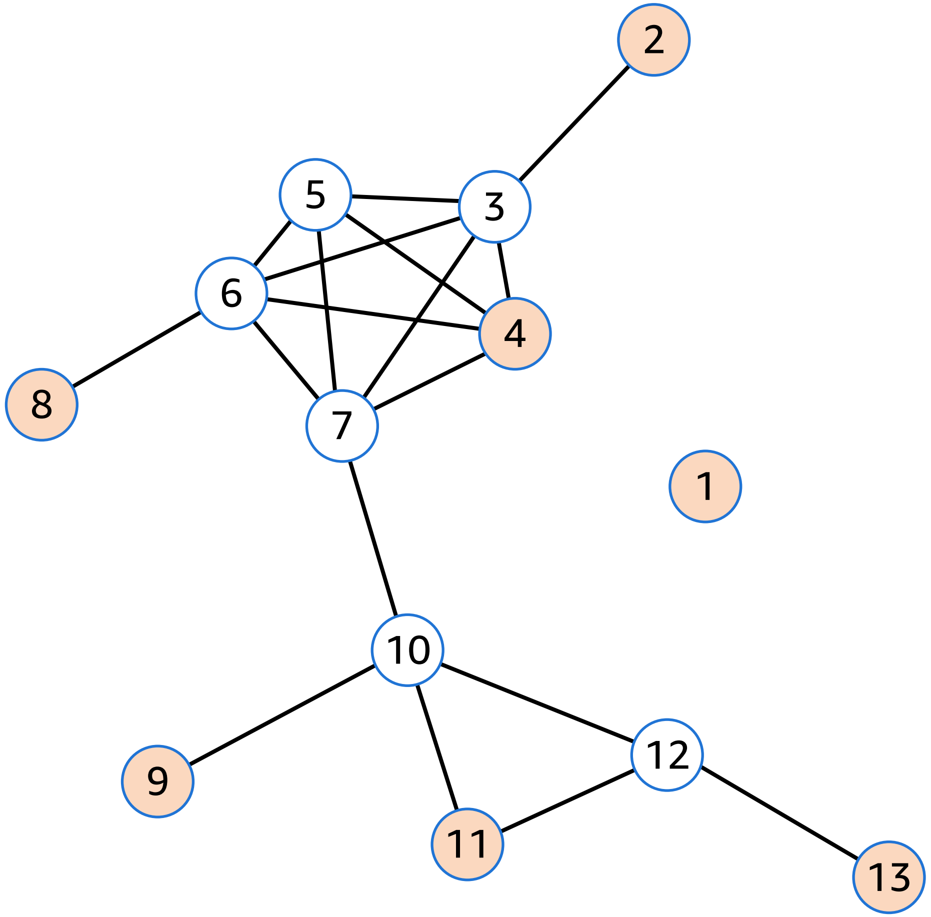
3:

4:

5:

Reduction/kernelization for the MIS problem

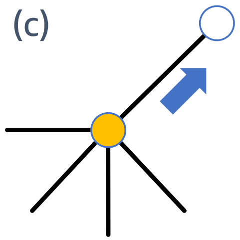


Observations:

- Graph fully reducible, with reduction factor $\xi = 100\%$.
- Reduction factor $\xi \in [0,1]$ for input graphs with N nodes and output with n nodes:

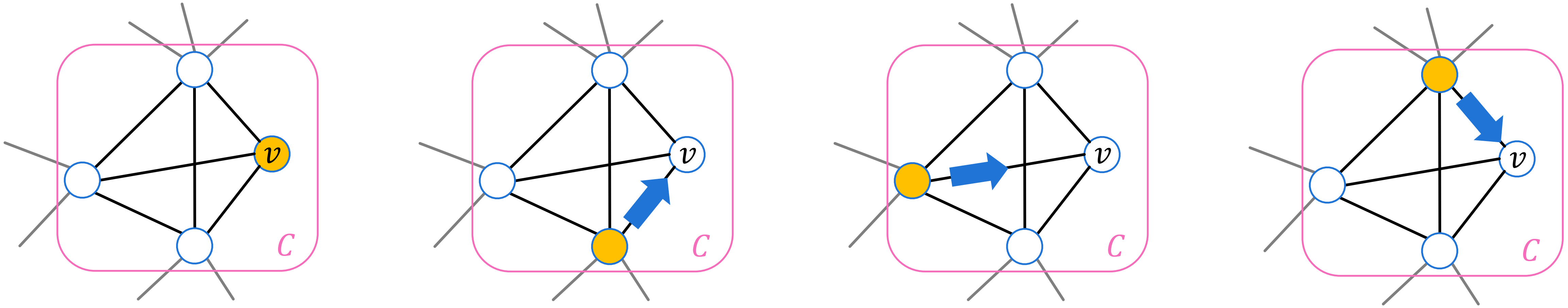
$$\xi = \frac{N - n}{N}$$

- Optimal MIS solution found by mere (clique-based) reduction.



Optimality by cut-and-paste

- Since v has no neighbors outside of the clique, by a *cut-and-paste* argument, it must be in some maximum independent set.
- Therefore, we can add v to the maximum independent set we are computing, and remove v and C from the graph.

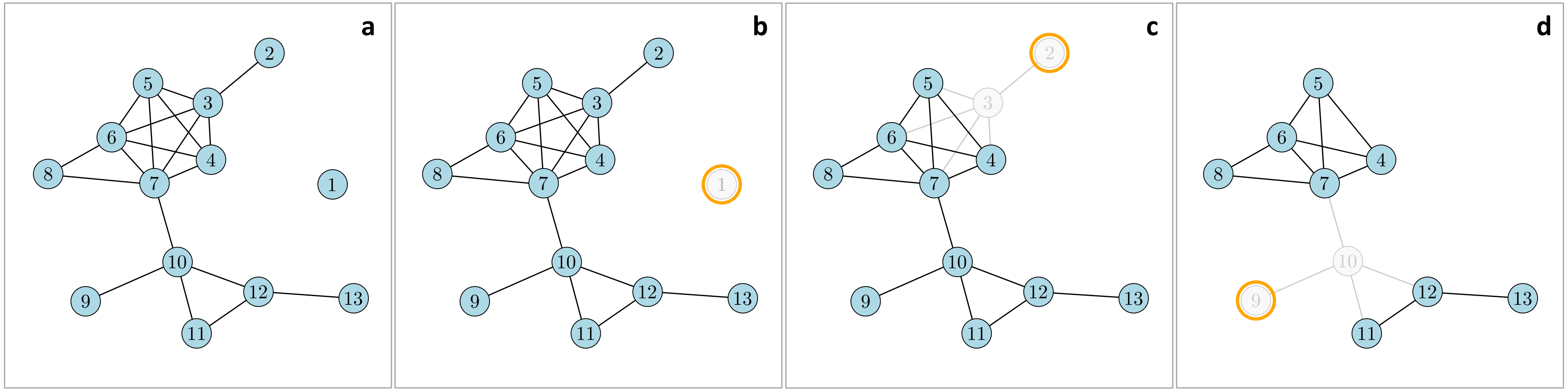


Reduction experiments

Reduction for real-world networks

network	input graph $\mathcal{G}$			kernel $\mathcal{K}$ of input graph $\mathcal{G}$				run time	reduction
	nodes	edges	average degree	nodes	edges	components	largest component	[ms]	ξ
Florentine	15	20	2.67	0	0	—	—	0.38	100%
Zachary Karate	34	78	4.59	4	4	1	4	0.97	88.2%
Dolphins	62	159	5.13	20	30	2	12	1.51	67.7%
Les Miserables	77	254	6.60	0	0	—	—	2.21	100%
Jazz	198	2742	27.70	83	580	1	83	17.74	58.1%
C. Elegans	438	1519	6.94	19	26	1	19	14.32	95.7%
Email	1133	5451	9.62	315	818	1	315	44.05	72.2%
Cora	2708	5278	3.90	79	94	16	9	116.93	97.1%
Citeseer	3264	4536	2.78	217	341	26	83	67.30	93.4%
PubMed	19714	44281	4.49	16	23	2	11	665.06	99.9%

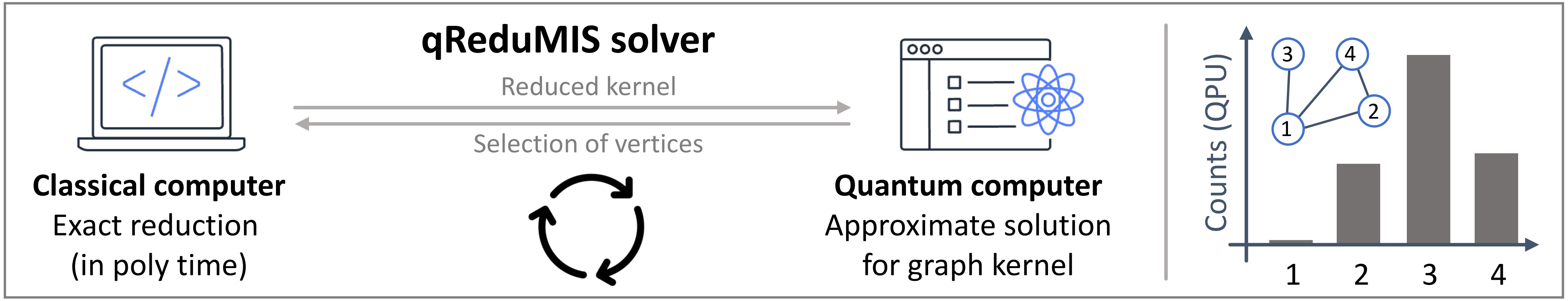
Preliminary summary – part II



- Classical reduction (kernelization) is a powerful tool for MIS/MWIS problems.
- Reduction methods are provably optimal.
- Reduction methods are fast (e.g., linear-time for sparse graphs).
- Reduction methods insensitive to the hardness parameter.

qReduMIS algorithm

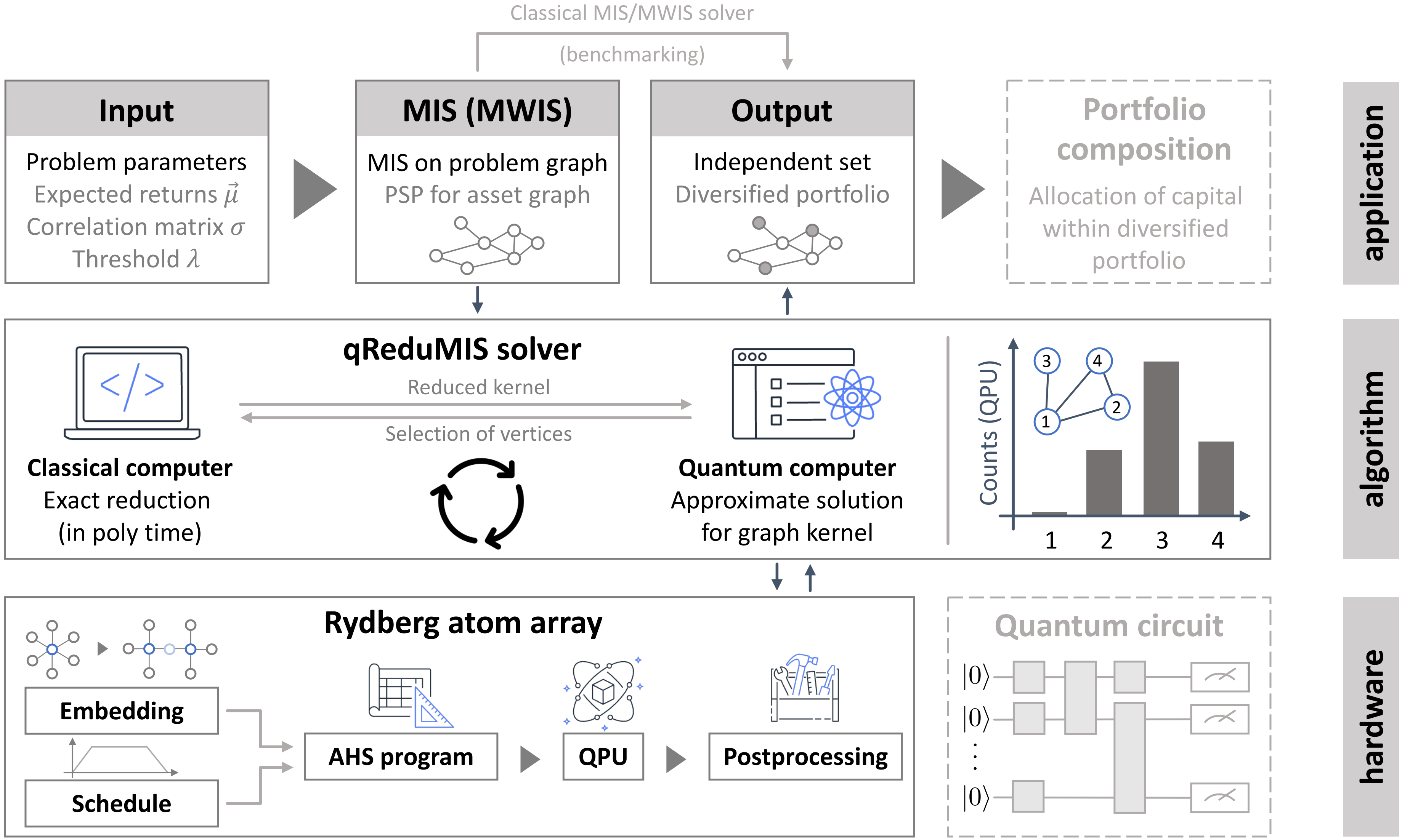
Motivation for qReduMIS



algorithm

- Exploit (exact and fast) reduction as much as possible, akin to classical SOTA heuristics.
- Reuse reduction to boost performance at the expense of (small) overhead.
- Periodically leverage QPU's native sampling capabilities (in the form of sample persistence) to unblock reduction whenever necessary.

The qReduMIS framework

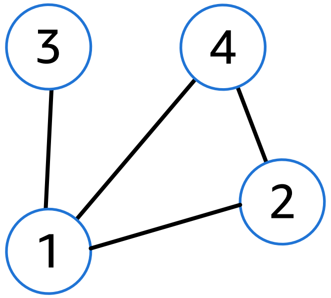


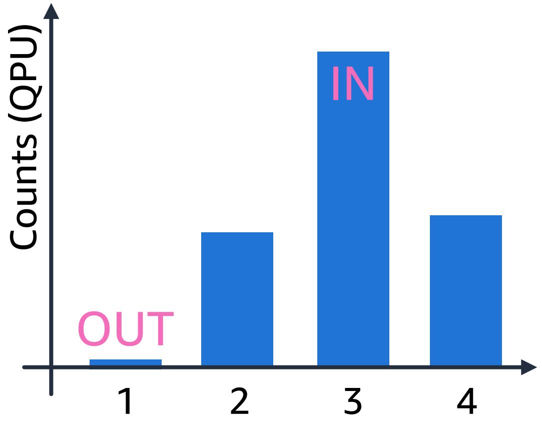
qReduMIS – High-level (pseudo-code) structure

```
Input graph G = (V, E), RCL size K_CL, selection parameter λ # default K_CL = λ = 1
Global W = ∅, S = ∅, R = ∅ # best solution, set of selected nodes, set of removed nodes
Procedure qReduMIS(G)
    if G empty return
    (K, s, r) ← ClassicalReduce(G) # get kernel, selected and removed nodes
    Append s to S, append r to R
    if K is empty then update and return W
    {I_n} ← QuantumMIS(K) # get intermediate independent sizes for kernel (incl. postprocessing) for
                          # all n = 1 …, N_shots shots
    if |S| + max{|I_n|} > |W| then update W # update incumbent
    Q, S, R ← Select ({I_n}, strategy = "in") # select λ frozen (fixed) degrees with "in"/"out" strategy
                                              # update sets S and R accordingly
    K' ← K[V_K\Q] # get inexact kernel
    qReduMIS(K') # recurse on inexact kernel
Return W
```

Quantum-informed identification of frozen nodes

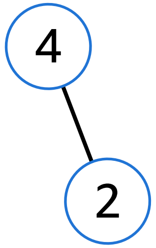
MIS={2, 3}
MIS={3, 4}



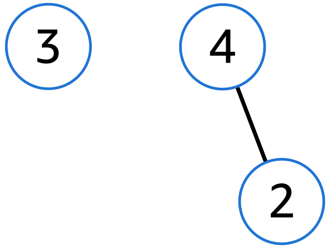


- Node 1 frozen
- Node 3 frozen

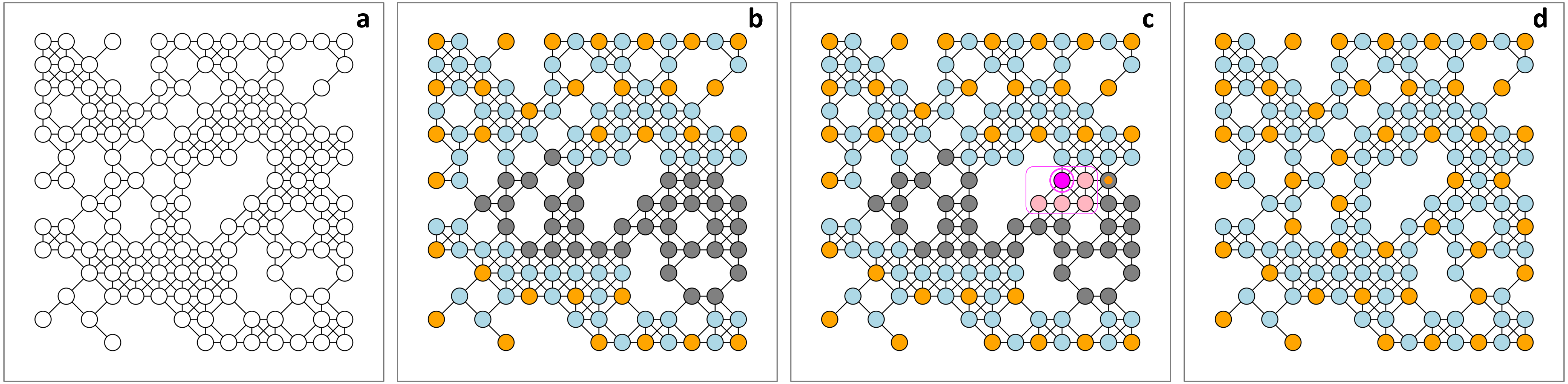
In-set strategy



Out-of-set strategy



Snapshots of the qReduMIS algorithm



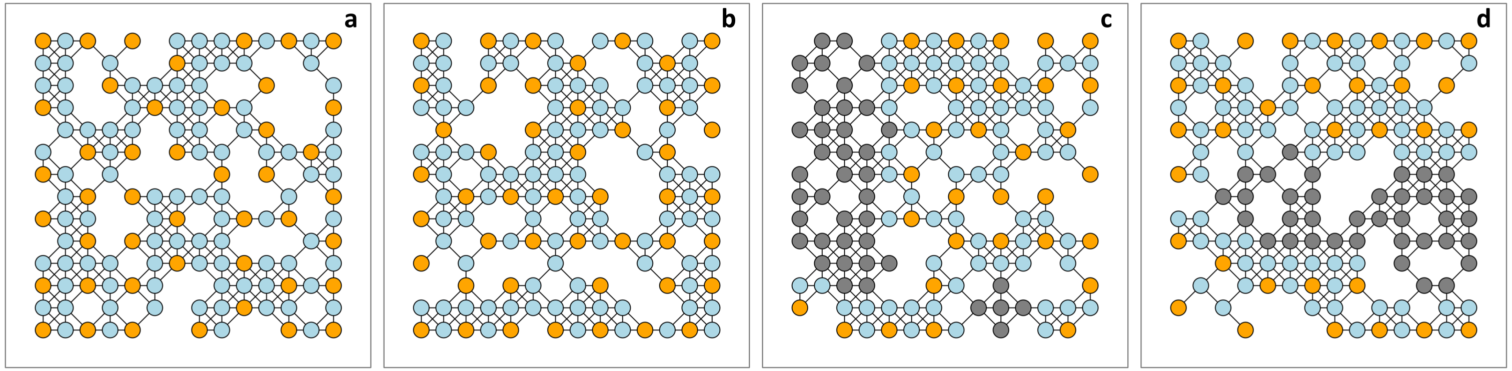
- Problem input: Site-diluted union-jack graph with 137 nodes.
- Hard instance with hardness ~1435.

- Classical reduction removes 100 nodes.
- Irreducible kernel (grey) found with 37 kernel nodes.

- QPU is called to unblock reduction.
- Node in pink identified as frozen node with high in-set probability.

- Remaining kernel fully reducible.
- MIS solution found with two classical reduction steps and one QPU call.

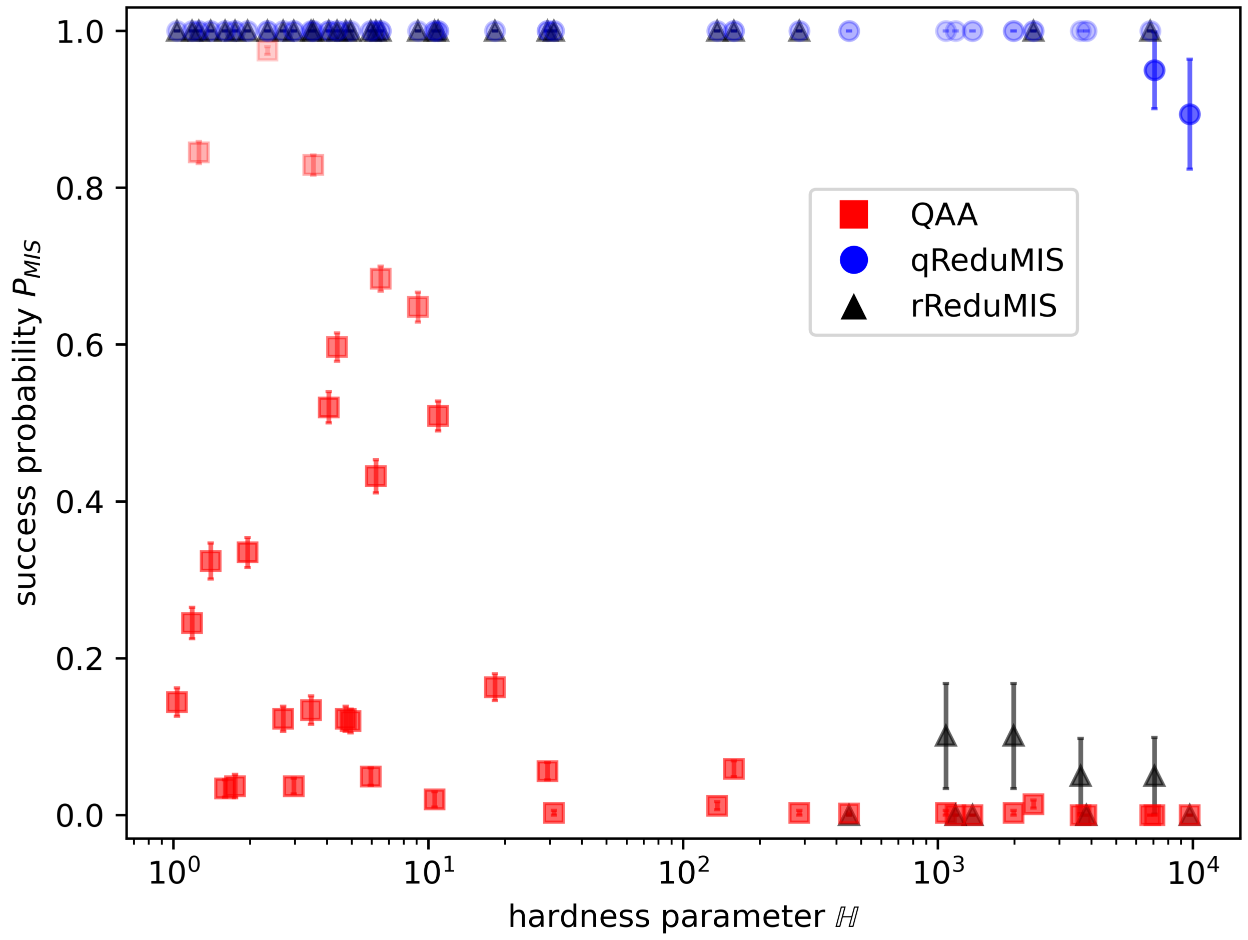
qReduMIS performance for example instances



Instance hardness $\mathbb{H}$	~ 1.478	~ 14.12	~ 125.5	~ 1435
QAA	35.2%	0.8%	0.7%	0%
QAA (Ref. [34])	47.8%	0.4%	1.9%	0%
qReduMIS	100%	100%	100%	100%

Average success probabilities P_{MIS} achieved with QAA and qReduMIS (with in-set selection strategy) using the QuEra Aquila QPU for four test instances (all with n = 137 vertices).

qReduMIS performance across larger testbed



Systematic experiments:

- Testbed with 39 random UJ instances with hardness ranging from ~ 1.03 up to ~ 9717.
- qReduMIS solves most instances to optimality with $P_{MIS} = 1$ and maintains a non-zero success rate with average $P_{MIS} \gtrsim 89\%$ throughout the testbed.

qReduMIS is hardware agnostic

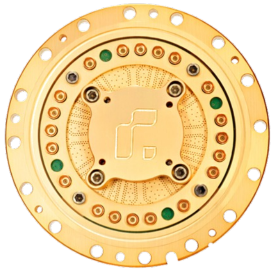
- The qReduMIS hardware layer can involve analog or digital quantum devices.
- **Task**: To unblock reduction, solve MIS problem on (kernel) graph:

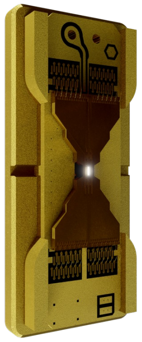
$$H(\mathbf{n}) = -\sum_{i \in \mathcal{V}_{\mathcal{K}}} n_i + U \sum_{(i,j) \in \mathcal{E}_{\mathcal{K}}} n_i n_j, \quad \blacktriangleright \quad \hat{H}_{\text{cost}} = \sum_{i,j} J_{ij} \hat{Z}_i \hat{Z}_j + \sum_i h_i \hat{Z}_i,$$

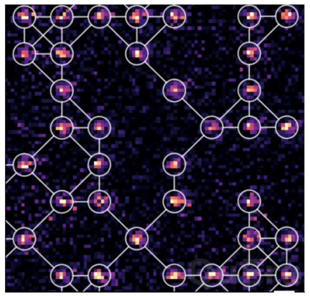
- **Approach**: Prepare low-energy state via optimized control parameters θ:

$$|\Psi(\theta)\rangle = \mathcal{U}(\theta) |\Psi_{\text{initial}}\rangle,$$

  - Gate-based QAOA (SC qubits, trapped ions, …).
  - Ising machines, including SC annealers (D-Wave).
  - AHS with Rydberg annealers (neutral atoms).
- **Impact** beyond Rydberg community…



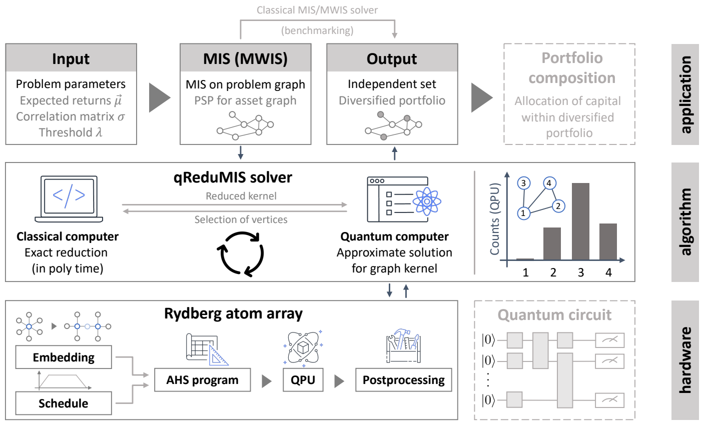


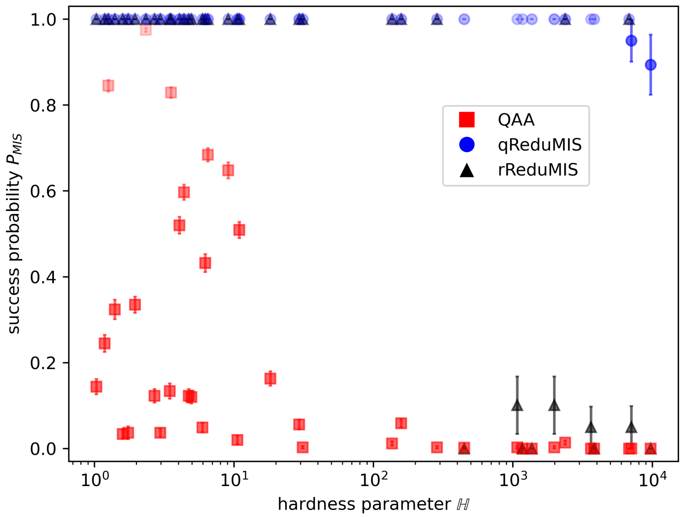


Summary and outlook

Summary and outlook

- Novel quantum-informed qReduMIS algorithm.
- Kernelization is applied in tandem with a quantum coprocessor that helps guide the repeated reduction process through the identification of frozen variables.
- Successful experiments with up to 231 qubits on QuEra, showing that qReduMIS can tackle fundamental performance limitations.
- **Outlook**: Future experiments based on alternative hardware platforms (such as superconducting qubits or trapped ions).





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Thank you.

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